АВТОМАТИКА
и
ТЕЛЕМЕХАНИКА

*Volume 19, No. 12*

*December 1958*



SOVIET INSTRUMENTATION AND
CONTROL TRANSLATION SERIES

# Automation and Remote Control

(The Soviet Journal *Avtomatika i Telemekhanika* in English Translation)

■ This translation of a Soviet journal on automatic control is published as a service to American science and industry. It is sponsored by the Instrument Society of America under a grant in aid from the National Science Foundation, continuing a program initiated by the Massachusetts Institute of Technology.

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Publication of *Avtomatika i Telemekhanika* in English translation started under the present auspices in April 1958 with Russian Vol. 18, No. 1 of January 1957. Translation of Vol. 18 has now been completed. The twelve issues of Vol. 19 will be published in English translation by mid-1959.

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Instrument Society of America
313 Sixth Avenue, Pittsburgh 22, Penna.

*Translated and printed by Consultants Bureau, Inc.*

Volume XIX No. 12 – December 1958
English Translation Published June 1959

# Automation and Remote Control

***The Soviet Journal Avtomatika i Telemekhanika in English Translation***

*Reported circulation of the Russian original 8,000.*

***Avtomatika i Telemekhanika is a Publication of the Academy of Sciences of the USSR***



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# A THEORY FOR DETERMINING OPTIMUM DYNAMIC SYSTEMS

N. I. Andreev

(Khar'kov)

**Sufficient conditions for an extreme of a certain functional, taken as a criterion for comparing various dynamic systems are derived.**

**Two examples illustrating the proposed technique for choosing an optimum dynamic system are given.**

It was shown in [1] that the problem of finding an optimum dynamic linear system that would ensure an extreme in functional I, which represents a given function n + 1 of quadratic* functionals of a unit pulse transfer function of system k (t), can be reduced to finding an extreme of the quadratic functional

$$I_{II} = \theta_1 I_1 [k(t)] + \ldots + \theta_n I_n [k(t)] + I_{n+1} [k(t)].$$

Function $k_\theta(t)$, which ensures an extreme in functional $I_{II}$, depends on the artificially introduced coefficients $\theta_i$. The values of coefficients $\theta_i = \theta_{i0}$ at which the extreme of functional $I[k(t)]$ is obtained can be determined by one of the two methods suggested in [1].

The required conditions which must be satisfied by $k^0(t)$ in order to obtain an extreme in functional I consist of the following:

1) $k^0(t) = k_\theta(t, \theta_{10}, \ldots, \theta_{n0})$ must be the extreme, i.e., must be the solution of Euler's equation corresponding to functional $I_{II}$;

2)
$$\left.\frac{\partial I [k^0(t)]}{\partial \theta_i}\right|_{\theta_j = \theta_{j0}} = 0, \quad i, j = 1, \ldots, n$$

or (a special case)

$$\frac{\partial I}{\partial I_l} = 0, \quad l = 1, \ldots, n+1.$$

In [1] the question of sufficient conditions which function $k^0(t)$ must satisfy in order to ensure an extreme in functional I was not raised.

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* It should be remembered that the term quadratic functional is applied to a functional $I_K$ when it possesses the following properties:

$$I_{\text{K}} [k(t) + \Delta\kappa(t)] = A_0 + A_1\Delta + A_2\Delta^2,$$

where $k(t)$ and $\kappa(t)$ are arbitrary functions from a class of permissible functions, and $A_0$, $A_1$, and $A_2$ are numbers related to functions $k(t)$ and $\kappa(t)$ but independent of $\Delta$.

The present article gives a generalization of the material presented in [1] and in addition the sufficient conditions for an extreme in functional $I = I[k(t)]$.

Let us formulate the posing of the problem.

Let us assume that functional $I[k(t)]$ represents a given differentiable function $\Phi$ of functionals $I_1[k(t)], \ldots, I_{n+1}[k(t)]$ (these functionals need not be quadratic):

$$I = \Phi(I_1, \ldots, I_{n+1}). \tag{1}$$

Let us also assume that there is a method of determining a function which would ensure an extreme in functional

$$I_1 = \theta_1 I_1[k(t)] + \cdots + \theta_n I_n[k(t)] + I_{n+1}[k(t)] \tag{2}$$

with any values of coefficient $\theta_i$ and that in particular the required and sufficient conditions which must be satisfied by function $k_0(t, \theta_1, \ldots, \theta_n)$ for ensuring an extreme in functional $I_I$ are known.

With these conditions it becomes necessary to state a method of determining function $k^0(t)$ which ensures an extreme in functional I, and to find in particular the required and sufficient conditions which function $k^0(t)$ must satisfy.

In order that function $k^0(t)$ should ensure an extreme in functional I it is necessary that the first variation of this functional with $k(t) = k^0(t)$ should equal zero:

$$\delta I = \frac{dI[k^0(t) + \Delta\kappa(t)]}{d\Delta}\bigg|_{\Delta=0} = 0, \tag{3}$$

where $\kappa(t)$ is any arbitrary function from a class of permissible functions.

Taking into account (1) let us rewrite condition (3) in the following form:

$$\frac{\partial\Phi}{\partial I_1}\frac{dI_1[k^0(t) + \Delta\kappa(t)]}{d\Delta}\bigg|_{\Delta=0} + \cdots + \frac{\partial\Phi}{\partial I_{n+1}}\frac{dI_{n+1}[k^0(t) + \Delta\kappa(t)]}{d\Delta}\bigg|_{\Delta=0} = 0 \tag{4}$$

The first variation of I, as it will be seen from expression (4), can equal zero if at least one of the following conditions is fulfilled:

1) $$\partial\Phi / \partial I_i = 0, \quad i = 1, \ldots, n+1; \tag{5}$$

2) if at least one derivative $\partial\Phi / \partial I_i$ does not equal zero (without reducing the generality let us assume that $\partial\Phi / \partial I_{n+1} \neq 0$) and if the following equation holds

$$a_1\frac{dI_1[k^0(t) + \Delta\kappa(t)]}{d\Delta}\bigg|_{\Delta=0} + \cdots + a_n\frac{dI_n[k^0(t) + \Delta\kappa(t)]}{d\Delta}\bigg|_{\Delta=0} + \\ + \frac{dI_{n+1}[k^0(t) + \Delta\kappa(t)]}{d\Delta}\bigg|_{\Delta=0} = 0, \tag{6}$$

where

$$a_i = \frac{\partial\Phi / \partial I_i}{\partial\Phi / \partial I_{n+1}}.$$

Equation (5) is, of course, a necessary condition for an extreme in function $\Phi(I_1, \ldots, I_{n+1})$ with certain values of arguments $I_i = I_{i0}$.

If with a certain function $k(t) = k_{00}(t)$ the functional $I_i$ assumes the values $I_i = I_{i0}$, it should be checked further whether the many variables function $\Phi$ satisfies the sufficient condition for an extreme with values $I_i = I_{i0}$. This sufficient condition is the inequality [2]

$$\sum_{i,j=1}^{n+1}\frac{\partial^2\Phi}{\partial I_i \partial I_j}\delta I_i \delta I_j > 0$$

or

$$\sum_{i,\,j=1}^{n+1} \frac{\partial^2 \Phi}{\partial I_i \partial I_j} \delta I_i \delta I_j < 0 \tag{7}$$

with arbitrary values of $\delta I_i$ and $\delta I_j$ variations.

If conditions (5) and (7) are fulfilled for a certain function $k_{00}(t)$, this function will ensure the extreme for functional I.

In problems arising from practical requirements condition (5) is not usually satisfied for the selected (given) class of function $k(t)$. Functional I becomes in such problems the criterion for comparing various systems (for instance systems of automatic control) and represents a smoothly changing function of functional $I_i$, moreover the extremes of function (1) are usually outside region G of the variations of functionals $I_i = I_i[k(t)]$.

We shall consider henceforth that in the region of variations of quantity $I_i$ [for a given class of function $k(t)$] all partial derivatives $\partial \Phi / \partial I_i$ cannot equal zero simultaneously. In this case, as it has already been pointed out, the required condition for an extreme in functional I is condition (6).

Equation (6) is a complex integro-differential equation; we shall solve it, therefore, in two stages. First we shall find a general form for the solution

$$\theta_1 \frac{dI_1[k_0(t) + \Delta \kappa(t)]}{d\Delta}\bigg|_{\Delta=0} + \ldots + \theta_n \frac{dI_n[k_0(t) + \Delta \kappa(t)]}{d\Delta}\bigg|_{\Delta=0} + \frac{dI_{n+1}[k_0(t)+\Delta\kappa(t)]}{d\Delta}\bigg|_{\Delta=0} = 0, \tag{6a}$$

where $\theta_i$ are as yet undetermined parameters. This solution $k_0(t)$ depends on parameters $\theta_i : k_0 = k_0(t, \theta_1, \ldots, \theta_n)$.

Next let us determine $\theta_i = \theta_{i0}$ in a way to satisfy condition

$$\frac{\partial \Phi / \partial I_i}{\partial \Phi / \partial I_{n+1}}\bigg|_{I_l = I_l[k_0(t,\,\theta_1,\,\ldots,\,\theta_n)]} = \theta_i, \quad i, l = 1, \ldots, n. \tag{8}$$

Such a splitting into two parts of the solution of equation (6) is possible because the solution of this complex equation coincides with the solution of equation (6a) if the coefficients of (6) are equal to

$$a_i = \theta_i, \quad i = 1, \ldots, n. \tag{9}$$

Hence, the solution of $k^0(t)$ lies in the class of functions $k_0(t, \theta_1, \ldots, \theta_n)$. The values of coefficients $\theta_{i0}$ with which condition (9) is satisfied are found from condition (8).

Equation (6a) is the necessary condition to obtain with $k(t) = k_0(t)$ the extreme of functional $I_l$ [see (2)]. According to the second condition of posing the problem the method of determining the general solution of equation (6a) is known. If this solution $k_0(t, \theta_1, \ldots, \theta_n)$ of equation (6a) can be determined in its final form, we shall obtain a system of algebraic or transcendental equations, determining the value of $\theta_{i0}$, by substituting expression $k_0(t, \theta_1, \ldots, \theta_n)$ in the left-hand sides of equations (8) or, which is the same thing, in equation (9). Usually under these conditions the solution of the system of equations (8) is not single-valued. In such cases the value of functional I should be calculated for coefficients $\theta_i$ which correspond to the variations in the solution of the system of equations (8) and those in the neighborhood of the variations, selecting from them the variation at which functional I has the largest (smallest) value.

In practice there can be cases when the solution $k_0(t, \theta_1, \ldots, \theta_n)$ in a certain region of variations of coefficients $\theta_i$ has one analytical expression and in another region of variations a different one. In such a case the solution of the system of equations (8) should be sought for each of these regions. Moreover, in one of the regions there may not exist an extreme for functional $I_I$. Such a region of $\theta_i$ parameters should be excluded from further consideration.

In searching for coefficients $\theta_{i0}$ which satisfy the system of equations (8) even greater difficulties may arise. Sometimes, it is more convenient to determine coefficients by another method. This method consists in substituting solution $k_0(t, \theta_1, \ldots, \theta_n)$ of equation (6a) into functional I. In this case functional I is converted into a function of $\underline{n}$ variables $\theta_1, \ldots, \theta_n$: $I = F(\theta_1, \ldots, \theta_n)$.

Values of coefficients $\theta_{i0}$ which correspond to the extreme of the functional I are determined in this case by the system of equations

$$\frac{\partial F(\theta_1, \ldots \theta_n)}{\partial \theta_i} = 0, \quad i = 1, \ldots, n. \tag{10}$$

Moreover, one or several of the equations (10) can be substituted by the condition (conditions) that $\frac{\partial F(\theta_1, \ldots, \theta_n)}{\partial \theta_i}$ does not exist.

The cases when $\partial F / \partial \theta_i$ do not exist are considered to be exceptional and will not be dealt with any further.

If a combination of parameters of $\theta_{i0}$ corresponds to the extreme of $I = I[k_0(t, \theta_1, \ldots, \theta_n)]$, it is the solution of the system of equations (8) and (10), since these systems of equations are the necessary conditions for an extreme in functional I.

If the combination of parameters $\theta_{i0}$, however, does not correspond to an extreme $I = I[k_0(t, \theta_1, \ldots, \theta_n)] = F(\theta_1, \ldots, \theta_n)$ but is the solution of the system of equations (8), it coincides with the solution of the system of equations (10) and, vice versa, the solution of the system of equations (10) coincides with the solution of the system of equations (8) with the condition that

$$\begin{vmatrix} \frac{\partial I_1}{\partial \theta_1}, & \ldots, & \frac{\partial I_n}{\partial \theta_1} \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ \frac{\partial I_1}{\partial \theta_n}, & \ldots, & \frac{\partial I_n}{\partial \theta_n} \end{vmatrix} \neq 0 \text{ for } \theta_i = {}_{i0}. \tag{11}$$

In order to prove this statement let us assume that condition (8) for function $k_0(t, \theta_1, \ldots, \theta_n)$ has been fulfilled with a certain combination of the values of parameters $\theta_i = \theta_{i0}$, $i = 1, \ldots, n$. Let us show that with the same combination of parameter values $\theta_i = \theta_{i0}$ condition (10) is being fulfilled for the same function $k_0(t, \theta_1, \ldots, \theta_n)$ which satisfies the necessary condition for an extreme in functional $I_I$. This necessary condition, as it was pointed out before, is the equality to zero of the first variation of the functional

$$I_I = \theta_{10} I[k(t)] + \cdots + \theta_{n0} I_n[k(t)] + I_{n+1}[k(t)]$$

when $k(t) = k_0(t, \theta_{10}, \ldots, \theta_{n0})$.

Hence, the conclusion that the partial derivatives of $I_I$ with respect to parameters $\theta_i$ with $\theta_j = \theta_{j0}$ $(i, j = 1, \ldots, n)$ are equal to zero

$$\left.\frac{\partial I_I}{\partial \theta_i}\right|_{\theta_j = \theta_{j0}} = \theta_{10} \left.\frac{\partial I_1[k_0]}{\partial \theta_i}\right|_{\theta_j = \theta_{j0}} + \cdots$$

$$\cdots + \theta_{n0} \left.\frac{\partial I_n[k_0]}{\partial \theta_i}\right|_{\theta_j = \theta_{j0}} + \left.\frac{\partial I_{n+1}[k_0]}{\partial \theta_i}\right|_{\theta_j = \theta_{j0}} = 0. \tag{12}$$

By substituting in relationships (12) coefficients $\theta_{i0}$ in front of the partial derivatives $\partial I_i / \partial \theta_i$ by the ratios of the partial derivatives (8) and then multiplying relationships (12) by $\partial \Phi / \partial I_{n+1}$ we obtain equations (10):

$$\frac{\partial \Phi}{\partial I_1} \frac{\partial I_1}{\partial \theta_i} + \cdots + \frac{\partial \Phi}{\partial I_{n+1}} \frac{\partial I_{n+1}}{\partial \theta_i} = \frac{\partial F}{\partial \theta_i} = 0, \tag{13}$$

where $k(t) = k_0(t, \theta_1, \ldots, \theta_n)$, $\theta_j = \theta_{j0}$, $i, j = 1, \ldots, n$.

Let us show that it holds conversely as well, if condition (10) is fulfilled by a certain function $k_0(t, \theta_1, \ldots, \theta_n)$ which is the solution of equation (6a) with parameter values $\theta_i = \theta_{i0}$, then these parameter values will turn equations (8) into identities.

Equation (10) can be written in an expanded form (13) or divided by the derivative $\partial \Phi / \partial I_{n+1}$ and written as

$$\frac{\partial \Phi / \partial I_1}{\partial \Phi / \partial I_{n+1}} \frac{\partial I_1}{\partial \theta_i} + \cdots + \frac{\partial \Phi / \partial I_n}{\partial \Phi / \partial I_{n+1}} \frac{\partial I_n}{\partial \theta_i} + \frac{\partial I_{n+1}}{\partial \theta_i} = 0, \tag{14}$$

$$\theta_j = \theta_{j0}, \quad i, j = 1, \ldots, n.$$

By considering that in this case equations (12) still hold, and comparing the systems of equations (12) and (14) it is possible to conclude [3] that if condition (11) is fulfilled the values of $\theta_i = \theta_{i0}$ also satisfy equations (8). In practical problems condition (11) is normally fulfilled. Hence, conditions (8) and (10) determine identical combinations of values $\theta_{i0}$, $i = 1, \ldots, n$.

For future reasoning it should be noted that equation (6a), which is the necessary condition for function $k^0(t)$, corresponding to an extreme for functional I, can be obtained from two considerations. Functional I of function $k(t)$ is also a function of $n + 1$ variables $I_i$, i.e., is a function of a point in space of $n + 1$ dimensions.

Region G of the $n + 1$ dimensional space, in which function $\Phi(I_1, \ldots, I_{n+1})$ is given, is determined by the limits in which the values of functionals $I_i[k(t)]$ vary with a given class of permissible functions $k(t)$. If function $\Phi(I_1, \ldots, I_{n+1})$ reaches an extreme within the region of variations of variables $I_i$, the values of variables $I_{i0}$ at which the extreme is reached are determined by conditions (5) and (7).

Other extremes of functional I corresponding to the largest or the smallest values of function $\Phi(I_1, \ldots, I_{n+1})$ can be obtained only on the boundaries of region G. Hence, in order to find these extremes of functional I it is necessary to find the boundary of region G and then to find the largest (smallest) value of function $\Phi$ on the boundary.

The boundary of region G can be determined the following way. Let us assume certain values for functionals $I_1, \ldots, I_n$:

$$I_1 = C_1, \ldots, I_n = C_n. \tag{15}$$

With these conditions let us determine the extremes of functional $I_{n+1} = I_{n+1}[k(t)]$. It will be seen from [4] that in this case the problem of determining function $k(t)$ which would ensure a conditional extreme for functional $I_{n+1}$ is reduced to finding a function which would ensure an absolute extreme for functional

$$I_I = \theta_1 I_1 + \cdots + \theta_n I_n + I_{n+1},$$

where $\theta_i$ are the undetermined parameters, whose values are found from conditions (15). The necessary condition for function $k_0(t, \theta_1, \ldots, \theta_n)$ to ensure an extreme for functional $I_I$ is equation (6a).

It should be noted that in this posing of the problem the values of $\theta_i$, which satisfy condition (15), are of no interest to us. It is important for us to know that functional $I_i$ attains its boundary values when $k(t) = k_0(t, \theta_1, \ldots, \theta_n)$ where function $k_0$ is the solution of equation (6a). Whence, we conclude that the boundary of region G may be determined by the relationship

$$I_i = I[k_0(t, \theta_1, \ldots, \theta_n)], \quad i = 1, \ldots, n + 1 \tag{16}$$

and represents an n-dimensional space. Functional I represents on this boundary a function of $\underline{n}$ variables $\theta_i$:

$$I = \Phi(I_1, \ldots I_{n+1}) = F(\theta_1, \ldots, \theta_n).$$

The necessary condition for an extreme of function F is written in the form of relationships (10).

The above new approach to the determination of function $k_0(t, \theta_1, \ldots, \theta_n)$ and the values of coefficients $\theta_i$ which correspond to the extreme of functional I are not related to the differentiability of function $\Phi(I_1, \ldots, I_{n+1})$ with respect to values of $I_i$. This approach to the problem can also be used when selecting an optimum nonlinear dynamic system which ensures an extreme for function $\Phi = \Phi(k_1, \ldots, k_m)$ where $k_i$ are values related to the characteristics of a nonlinear dynamic system. The suggested approach to the solution of the problem of finding an extreme for functional I is more convenient when sufficient conditions for the extreme of this functional are found. In this connection let us examine two cases.

In the first case, when the extreme of $\Phi(I_1, \ldots, I_{n+1})$ is reached inside region G of the variation of functionals $I_i$, the sufficient conditions for the extreme are represented by relationships (5) and (7).

In the second case let us assume in addition that $\Phi(I_1, \ldots, I_{n+1})$ is a monotonically decreasing function of $I_i$ in region G and the boundary of region G is a convex surface, and then split the problem into two stages. In the first stage we shall obtain sufficient conditions for ensuring that with $k(t) = k_0(t, \theta_1, \ldots, \theta_n)$ where $[\theta_i - \theta_{i0}] < \epsilon_i$, $\epsilon_i$ being positive numbers, points $(I_1, \ldots, I_{n+1})$ lie on the region G boundary. In the second stage let us obtain sufficient conditions for function $F(\theta_1, \ldots, \theta_n) = I[k_0(\theta_1, \ldots, \theta_n)]$ to reach the extreme on the boundary of region G with certain values of parameters $\theta_i = \theta_{i0}$.

With $k(t) = k_0(t, \theta_1, \ldots, \theta_n)$ where $[\theta_i - \theta_{i0}] <$ $_i$ points $(I_1, \ldots, I_{n+1})$ lie on the region G boundary if functional $I_{n+1}$ reaches the extreme under conditions that $I_i = C_i$ where $i = 1, \ldots, n$; and $d_{i_1} < C_i < d_{i_2}$ $d_{i_1}$ and $d_{i_2}$ being numbers determining the region of number $C_i$ variations. The numbers $d_{i_1}$ and $d_{i_2}$ are functions of $\theta_{i0}$ and $\epsilon_i$. Hence, in order to ascertain that with $k(t) = k_0(t, \theta_1, \ldots, \theta_n)$ points $(I_1, \ldots, I_{n+1})$ lie on region G boundary it is necessary to have sufficient conditions for functional $I_I$ to reach the extreme with $k(t) = k_0(t, \theta_1, \ldots, \theta_n)$ where $[\theta_i - \theta_{i0}] < \epsilon_i$. When formulating the posing of the problem at the beginning of the paper, it was assumed that these sufficient conditions for the extreme of functional $I_I$ are known.

Now the problem of the second stage must be solved, i.e., the sufficient conditions for function $F(\theta_1, \ldots, \theta_n) = I[k_0]$ to reach the extreme at $\theta_i = \theta_{i0}$ must be found. Such conditions have already been written for function $\Phi(I_1, \ldots, I_{n+1})$. These conditions (5) and (7) assume the following form for function F

$$\sum_{i,\, j=1}^{n} \frac{\partial^2 F}{\partial\theta_i \partial\theta_j} \delta\theta_i \delta\theta_j > 0 \quad \frac{\partial F}{\partial\theta_i} = 0, \quad i = 1, \ldots, n, \tag{17a}$$

(17b)

or

$$\sum_{i,\, j=1}^{n} \frac{\partial^2 F}{\partial\theta_i \partial\theta_j} \delta\theta_i \delta\theta_j < 0, \quad i, j = 1, \ldots, n, \tag{17c}$$

where the partial derivatives are calculated with parameter values $\theta_e = \theta_{e0}$, $e = 1, \ldots, n$, which satisfy conditions (17a), and one of the conditions (17b) must be fulfilled with any values of parameter $\delta\theta_i$ and $\delta\theta_j$ variations.

Thus, the sufficient conditions for an extreme of the functional consist of sufficient conditions for an extreme of functional $I_I$ and conditions (17a) and (17b). The more complex conditions are usually those for the extreme of functional $I_I$. These conditions depend on the form of functional $I_I$.

Let us now give an example often encountered in practice when selecting optimum linear systems of automatic control. Let functional $I_I$ be quadratic of the form

$$I_1 = I_T = \int_0^T\int_0^T R(\lambda-\tau)\,k(\lambda)\,k(\tau)\,d\lambda\,d\tau + $$
$$+\sum_{i=0}^{m} C_i\left[\int_0^T \lambda^i k(\lambda)\,d\lambda\right]^2 + 2\sum_{j=0}^{m} D_j\int_0^T \lambda^j k(\lambda)\,d\lambda, \tag{18}$$

where $R(t)$ is the correlation function of a stationary continuous random process, $k(t)$ the integrated function in the interval $0 - T$, and $C_i$, $D_j$ are some numbers.

The necessary condition for an extreme (in this case a minimum) for functional $I_T$ can be obtained by means of making the first variation of $I_T$ equal to zero. In this concrete example it is easy to see that this condition has the form:

$$\int_0^T R(t-\tau)\,k(\tau)\,d\tau + \sum_{i=0}^{m}\left[C_i\int_0^T \tau^i k(\tau)\,d\tau + D_i\right]t^i = 0. \tag{19}$$

Let us show that this necessary condition for the minimum of functional $I_T$ is also a sufficient condition if $C_i \geq 0$ where $i = 1, \ldots, m$. For this purpose let us calculate the difference

$$I_T[k^0(t) + \varkappa(t)] - I[k^0(t)],$$

where $k^0(t)$ is a function satisfying condition (19), i.e., an extreme, $\kappa(t)$ an arbitrary function integrated over the time interval $0 - T$.

It is easy to see that

$$I_T[k^0(t)+\varkappa(t)] - I[k^0(t)] = 2\int_0^T\left\{\int_0^T R(\lambda-\tau)\,k(\tau)\,d\tau + \right.$$
$$\left. + \sum_{i=0}^{m}\left[C_i\int_0^T \tau^i k(\tau)\,d\tau + D_i\right]\lambda^i\right\}\varkappa(\lambda)\,d\lambda +$$
$$+\int_0^T\int_0^T R(\lambda-\tau)\,\varkappa(\lambda)\,\varkappa(\tau)\,d\lambda\,d\tau + \sum_{i=0}^{m} C_i\left[\int_0^T \varkappa(\lambda)\,\lambda^i\,d\lambda\right]^2 =$$
$$= \int_0^T\int_0^T R(\lambda-\tau)\,\varkappa(\lambda)\,\varkappa(\tau)\,d\lambda\,d\tau + \sum_{i=0}^{m} C_i\left[\int_0^T \lambda^i\varkappa(\lambda)\,d\lambda\right]^2 \geqslant 0, \tag{20}$$

since the first term is a variance of a random process, and the second a sum of quantities which are not negative. Thus, the sufficiency of conditions (19) have been proved. If some of the coefficients $C_i < 0$, however, condition (19) becomes insufficient in a general case. In practice the coefficients of $C_i$ are usually positive.

The above method is also applicable when function $k$ has two variables $k = k(t, \tau)$ and $I$ is an operator: $I = I[k(t, \tau)] = I(t)$. In this case variable $t$ should be considered a parameter. In this paper as in [1] for simplifying calculations parameter $t$ has been omitted.

In conclusion, we give two examples which illustrate the technique and basic principles of this paper and of [1].

Example 1. Let a signal $y$, consisting of a useful signal $x$ and interference $z$, be impressed on the input of a linear automatic control system. The useful signal $x$ can be represented in the form

$$x = x_0 + vt,$$

where $x_0$ is a random quantity with a large variance, $v$ is a given quantity and $t$ is time.

The interference is a stationary random process subject to the normal distribution law, with the following probability characteristics

mathematical expectation $M[z] = 0$,

correlation function $R_z(\tau) = e^{-\alpha|\tau|}$.

Quantities $x$ and $z$ are not interrelated, i.e., $M[xz] = 0$.

The process at the output of the automatic control system is represented by expression

$$\beta(t) = \int_0^T k(\tau)\, y(t-\tau)\, d\tau,$$

where $\beta(t)$ is the law of change of the system output coordinate; $k(t)$ the unit pulse transfer function of this system with $0 \le t \le T$, $k(t) = 0$ when $t < 0$ and $t > T$.

Let us assume that function $k(t)$ is subject to the following limitations.

$k(t)$ is the integrated function

$$\int_0^T k(\tau)\, d\tau = 1. \tag{21}$$

It is required to determine the unit pulse transfer function $k(t)$ at which the maximum probability is reached that the error of extrapolation

$$\Delta = x(t+T_e) - \beta(t) = x(t+T_e) - \int_0^T y(t-\tau)\, k(\tau)\, d\tau \tag{22}$$

will not exceed in its modulus a certain small quantity $\epsilon$. Here $T_e$ is a given extrapolation time.

The probability that $|\Delta| < \epsilon$, where $\epsilon$ is a small quantity, can be determined with great accuracy by the following approximate formula:

$$P_\epsilon = P_\epsilon(|\Delta| < \epsilon) = \frac{2\epsilon}{\sqrt{2\pi}\,\sigma} e^{-\frac{a^2}{2\sigma^2}}, \tag{23}$$

where $a = M[\Delta]$ is the mathematical expectation of the error of extrapolation, $\sigma^2 = M[(\Delta - a)^2]$ is the variance of the extrapolation error.

Henceforth, we shall not operate with the probability $P_\epsilon$, but with quantity $P$, proportional to $P_\epsilon$:

$$P = \frac{\sqrt{2\pi}}{2\epsilon} P_\epsilon = \frac{1}{\sigma} e^{-\frac{a^2}{2\sigma^2}}. \tag{23a}$$

It will be easily seen that by changing from $P_\epsilon$ to $P$ no substantial change has been introduced.

Quantities $a^2$ and $\sigma^2$ are functionals of function $k(t)$

$$a^2 - (M[\Delta])^2 = \left\{M\left[x(t+T_e) - \int_0^T y(t-\tau)\, k(\tau)\, d\tau\right]\right\}^2 = v^2\left[T_e + \int_0^T \tau k(\tau)\, d\tau\right], \tag{24}$$

$$\sigma^2 = M[(\Delta - a)^2] = M\left[\left\{\int_0^T z(t-\tau)\, k(\tau)\, d\tau\right\}^2\right] = \int_0^T\int_0^T R_z(\lambda - \tau)\, k(\lambda)\, k(\tau)\, d\lambda\, d\tau. \tag{25}$$

In this example functional I corresponds to quantity P, functional $I_2$ to functional $a^2$, functional $I_1$ to functional $\sigma^2$. Functional $I_I$ is in this case a quadratic functional, since $a^2$ and $\sigma^2$ are quadratic functionals. Hence, in this case functional $I_I$ has the form:

$$I_I = 0\sigma^2 + a^2. \tag{26}$$

Taking into account (24) - (26) we can write:

$$I_I = 0\int_0^T\int_0^T R_z(\lambda-\tau)\,k(\lambda)\,k(\tau)\,d\lambda\,d\tau + v^2\left[T_0 + \int_0^T \tau k(\tau)\,d\tau\right]^2.$$

Function $k(t) = k_\theta(t, \theta)$ which ensures an extreme (minimum) for functional $I_I$, with $\theta > 0$ and condition (21), ensures an absolute extreme for functional

$$\bar{I} = \theta\int_0^T\int_0^T R_z(\lambda-\tau)\,k(\lambda)\,k(\tau)\,d\lambda\,d\tau + v^2\left[T_0 + \int_0^T \tau k(\tau)\,d\tau\right]^2 + \gamma_0\int_0^T k(\tau)\,d\tau, \tag{27}$$

where $\gamma_\theta$ is an undetermined factor found from condition (21).

A necessary and sufficient condition for function $k_\theta(t, \theta)$ which ensures a minimum for functional $\bar{I}$ is the integral equation [5, 6]

$$\theta\int_0^T R_z(t-\tau)\,k(\tau)\,d\tau + v^2 t\int_0^T \tau k(\tau)\,d\tau + v^2 T_0\, t - \gamma_0 = 0, \tag{28}$$

which is a particular case of equation (6a).

The solution of this equation from [7] has with $R_z(\tau) = e^{-\alpha|\tau|}$ the form:

$$k_\theta(t, \theta) = A_0 + A_1 t + C_1\delta(t) + D_1\delta(t-T), \tag{29}$$

where $\delta$ stands for a delta function.

Coefficients $A_0$, $A_1$, $C_1$, and $D_1$ are determined by a system of equations obtained after substituting (29) in (28):

$$\theta\int_0^T e^{-\alpha|t-\tau|}\left[A_0 + A_1\tau + C_1\delta(\tau) + D_1\delta(\tau-T)\right]d\tau +$$

$$+ v^2 t\int_0^T \tau\left[A_0 + A_1\tau + C_1\delta(\tau) + D_1\delta(\tau-T)\right]d\tau + v^2 T_0 t - \gamma_0 = 0.$$

After integration, having equated to zero coefficients of $t^0$, $t$, $e^{-\alpha t}$, $e^{\alpha t}$, and using condition (21) we obtain five equations for the determination of five unknowns $A_0$, $A_1$, $C_1$, $D_1$, and $\gamma_0$:

$$\begin{aligned}
&\alpha A_0 + (\alpha T + 1)A_1 - \alpha^2 D_1 = 0,\\
&\alpha A_0 - A_1 - \alpha^2 C_1 = 0,\\
&T A_0 + \frac{T^2}{2}A_1 + C_1 + D_1 = 0,\\
&\frac{T}{2}A_0 + \left(\frac{T^3}{3} + \frac{20}{\alpha v^2}\right)A_1 + TD_1 = -T_0,\\
&\frac{20}{\alpha}A_0 - \gamma_0 = 0,
\end{aligned} \tag{30}$$

Hence

$$A_0=\frac{\alpha^2\left[(\alpha T+1)T+\left(\frac{T^3}{3}+\frac{2\theta}{\alpha v^2}\right)\alpha^2\right]+\left(\alpha T+\frac{\alpha^2T^2}{2}\right)\alpha^2T_e}{D_0},\tag{31}$$

$$A_1=-\frac{\alpha^2\left(\alpha T+\frac{\alpha^2T^2}{2}\right)+\alpha^3\quad(\alpha T+2)T_e}{D_0},$$

$$\gamma_0=\frac{2\theta}{\alpha}A_0,$$

where $D_0$ is a determinant consisting of the coefficients of the first four equations of system (30) and taken with a reversed sign, i.e.,

$$D_0=4\theta\frac{\alpha}{v^2}+2\alpha T\left(\theta\frac{\alpha^2}{v^2}+1\right)+2\alpha^2T^2+\frac{2}{3}\alpha^3T^3+\frac{\alpha^4T}{12}.\tag{32}$$

We shall now require to express the integral $\nu=\int_0^T\tau k(\tau)\,d\tau$ in terms of the initial values of the problem.

Omitting intermediate calculations we obtain

$$\nu=\int_0^T\tau k_0(\tau,\theta)\,d\tau=-\frac{2\theta}{\alpha v^2}A_1-T_e.\tag{33}$$

In order to fulfill the second stage of the problem it is necessary to express $a^2$ and $\sigma^2$ in terms of the initial values and coefficient $\theta$.

By using formulas (24), (31) - (33), we obtain

$$a^2(\theta)=v^2[T_e+\nu]^2=v^2\left(\frac{2\theta}{\alpha v^2}A_1\right)^2=$$

$$=4\frac{\alpha^2}{v^2}\theta^2\frac{(\alpha T+2)^2\left(\frac{\alpha T}{2}+\alpha T_e\right)^2}{\left[4\theta\frac{\alpha^2}{v^2}+2\alpha T\left(\theta\frac{\alpha^2}{v^2}+1\right)+2\alpha^2T^2+\frac{2}{3}\alpha^3T^3+\frac{\alpha^4T^4}{12}\right]^2}.\tag{34}$$

Using formulas (25), (28), (31) - (33) we obtain

$$\sigma^2(\theta)=\frac{\gamma_0}{\theta}-\frac{v^2}{\theta}(T_e+\nu)\nu=2\frac{A_0}{\alpha}-\frac{2}{\alpha}A_1\left(\frac{2\theta}{dv^2}A_1+T_e\right)=$$

$$=2\alpha\frac{(\alpha T+1)T+\alpha^2\left(\frac{T^3}{3}+\frac{2\theta}{\alpha v^2}\right)+\alpha T\left(1+\frac{\alpha T}{2}\right)T_e}{D_0}+$$

$$+\frac{2\alpha^3\left[T\left(1+\frac{\alpha T}{2}\right)+(\alpha T+2)T_e\right]}{D_0}\times\left[2\alpha\frac{\theta}{v^2}\frac{-\alpha T\left(1+\frac{\alpha T}{2}\right)-\alpha(\alpha T+2)T_e}{D_0}+T_e\right].$$

Introducing notations $\kappa=\alpha T$, $\kappa_e=\alpha T_e$, $\gamma^2=\frac{\alpha^2}{v^2}$, after transformation, we obtain

$$\sigma^2(\theta)=\frac{2}{D_0^2}\Big\{4\theta^2(2+\kappa)\gamma^4+4\left(2+2\kappa+\frac{2}{3}\kappa^2+\frac{1}{12}\kappa^3\right)\kappa\theta\gamma^2+$$

$$+\kappa\left(2+2\kappa+\frac{2}{3}\kappa^2+\frac{1}{12}\kappa^3\right)\left[\kappa\left(1+\kappa+\frac{\kappa^2}{3}\right)+(2+\kappa)\kappa_e(\kappa_e+\kappa_\nu)\right]\Big\}.\tag{35}$$

With these notation expression (34) assumes the form

$$a^2(\theta) = 4\gamma^2 \frac{(\kappa+2)^2\left(\frac{\kappa}{2}+\kappa_e\right)^2}{D_0}\theta^2,$$

where

$$D_0 = 2\gamma^2(2+\kappa)\theta + 2\kappa + 2\kappa^2 + \frac{2}{3}\kappa^3 + \frac{\kappa^4}{12}.$$

In subsequent calculations, we assume

$$\kappa = 3, \quad \kappa = 6, \quad \gamma^2 = 20. \tag{36}$$

With these concrete numerical values of the initial values the quantities $a^2$ and $\sigma^2$ can be expressed in terms of parameter $\theta$ as follows:

$$a^2(\theta) = \frac{2.8\,\theta^2}{(\theta+0.244)^2}, \quad \sigma^2(\theta) = \frac{0.40\,\theta^2 + 0.385\,\theta + 0.65}{(\theta+0.244)^2}. \tag{37}$$

According to the proposed method it is now necessary to find the value of parameter $\theta = \theta_0$, which solves equation (8). In this case (8) has the form:

$$\frac{\partial P/\partial\sigma^2}{\partial P/\partial a^2} = \theta$$

or

$$1 - \frac{a^2(\theta)}{\sigma^2(\theta)} = \theta. \tag{38}$$

Substituting in equation (38) expressions (37), we obtain the relationship

$$\frac{2{,}8\,\theta^2}{0{,}40\,\theta^2 + 0{,}385\,\theta + 0{,}65} = 1 - \theta,$$

which after transformation has the form:

$$\theta^3 + 7\theta^2 + 0.66\,\theta - 1.63 = 0. \tag{39}$$

From Descarte's theorem [3] it follows that equation (39) has one positive root. The other two roots are negative. These roots are not taken into account since it can be shown that with negative values of parameter $\theta$ the sufficient condition for a minimum in functional $I_I$ [see (20)] is not fulfilled, and that a maximum in functional $I = P$ is not reached either.

Before attempting to find the positive values of root $\theta_0$, of equation (39), let us note that this root is smaller than one, since it is only with $\theta < 1$ that the mathematical expectation $a(\theta)$ decreases and a rise in functional P can be obtained. Hence, the root of equation (39) should be sought in the interval 0 to 1. The value of this root is equal to $\theta_0 = 0.42$ with an accuracy of 0.01.

We do not propose to check the conditions for a maximum in functional P, instead let us determine the maximum for $P = P(\theta)$, by plotting the graph of this function.

By using formulas (23) and (37) $a^2$, $\sigma$, P, and $P^2$ were calculated for several values of $\theta$ and the table given below was compiled.

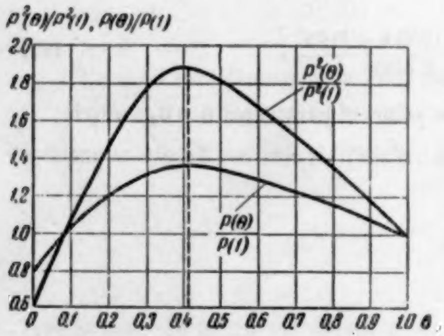
In addition to the values of $P(\theta)$ and $P^2(\theta)$ the table also shows the ratios between these values and $P(1)$ and $P^2(1)$ which correspond to the optimum system of automatic control, determined by the criterion of the minimum mean-square value of the error.

Functional $P^2$ has a completely definite physical meaning. By means of functional $\frac{\varepsilon^2}{\pi} P^2 = \frac{\varepsilon^2}{\pi\sigma^2} e^{-\frac{a^2}{\sigma^2}}$

it is possible to calculate with great accuracy the probability that $|\Delta_1| < \epsilon$ and $|\Delta_2| < \epsilon$, where $\epsilon$ is a small quantity, and $\Delta_1$ and $\Delta_2$ are errors in two similar automatic control systems which displace the final control element in mutually perpendicular directions, when independent from each other signals with similar probability characteristics are impressed on the system inputs.

| $\theta$ | 1.00 | 0.80 | 0.60 | 0.50 | 0.42 | 0.35 | 0.20 | 0.00 |
|---|---|---|---|---|---|---|---|---|
| $P^2(\theta)$ | 0.155 | 0.210 | 0.258 | 0.285 | 0.295 | 0.290 | 0.230 | 0.100 |
| $P^2(\theta)/P^2(1)$ | 1.00 | 1.35 | 1.66 | 1.83 | 1.90 | 1.86 | 1.48 | 0.65 |
| $P(\theta)$ | 0.395 | 0.458 | 0.508 | 0.534 | 0.542 | 0.538 | 0.480 | 0.315 |
| $P(\theta)/P(1)$ | 1.00 | 1.16 | 1.29 | 1.35 | 1.38 | 1.36 | 1.22 | 0.805 |

Data graphs showing $P^2(\theta)/P^2(1)$ and $P(\theta)/P(1)$ were plotted from the table (see drawing). It will be seen from the graphs that with $k_0(t, \theta)$, where $\theta = \theta_0 = 0.42$, the functionals $P^2$ and $P$ do attain their maxima.




A study of the table and the graphs leads to the conclusion that the selection of an optimum system in this example by means of the maximum P (or $P^2$) criterion leads to higher results than the selection by means of the minimum mean-square error criterion. The former gives a P 38% greater than the latter. The increase in $P^2$ is equal to 90%.

It is possible to evaluate functional $P^2$ for an optimum system selected by means of the maximum $P^2$ criterion, if the values of $a_0^2$ and $\sigma_0^2$ for an optimum system selected by the minimum mean-square error criterion are known. The values of $a_0^2$ and $\sigma_0^2$ correspond to the value $P^2 = P_0^2 = \frac{1}{\sigma_0^2} e^{-\frac{a_0^2}{\sigma_0^2}}$.

Certain relations $a_{00}^2 \leq a_0^2$ and $\sigma_{00}^2 \geq \sigma_0^2$ when $a_{00}^2 + \sigma_{00}^2 \geq a_0^2 + \sigma_0^2$ correspond to the optimum system selected by the $P^2$ criterion. Let us determine maximum $P^2$ under condition that $a^2$ and $\sigma^2$ can assume any value and that $a^2 + \sigma^2 = a_0^2 + \sigma_0^2$. It is easy to see that with this condition the $P^2$ maximum equals $\frac{1}{\sigma_0^2 + a_0^2}$.

Hence, the value of the functional $P^2 = P_{00}^2$, which corresponds to the optimum system selected by the $P^2$ criterion is smaller than $\frac{1}{\sigma_0^2 + a_0^2}$, i.e., $P_{00}^2 \leq \frac{1}{\sigma_0^2 + a_0^2}$.

Hence, if ratio $\frac{a_0^2}{\sigma_0^2}$ is small the difference $P_{00}^2 - P_0^2$ is also small.

It is also possible to evaluate $P_{00}^2$ from below if $a_0^2 > \sigma_0^2$. For this purpose let us reason as follows. In the transition from $P_0^2$ to $P_{00}^2$, $a^2$ decreases and $\sigma^2$ increases. The values of $a^2$ and $\sigma^2$ change continuously. In the process of their variation $a^2$ and $\sigma^2$ attain values of $a_{00}^2$ and $\sigma_{00}^2$ at which quantity $P^2$ is at a maximum.

Let us assume that $a^2$ remains constant and equal to $a_0^2$ and that $\sigma^2$ increases. With this condition $P^2$ reaches a maximum when $\sigma^2 = a_0^2$. This $P^2$ maximum is equal to $\frac{1}{a_0^2 e}$, hence $P_{00}^2 \geq \frac{1}{e a_0^2}$. This means that if the difference $\frac{1}{e a_0^2} - \frac{1}{\sigma_0^2} e^{-\frac{a_0^2}{\sigma_0^2}}$ is substantial, the changing of the criterion from the minimum mean-square error to the maximum $P^2$ gives a substantial gain in $P^2$.

For estimating the gain in $P^2$ it is sometimes more convenient to use the ratio $P_{00}^2/P_0^2$.

$$\frac{P_{00}^2}{P_0^2} \leqslant \frac{e^{\frac{a_0^2}{\sigma_0^2}}}{1+\frac{a_0^2}{\sigma_0^2}} \quad \text{at any} \quad a_0^2/\sigma_0^2, \tag{40}$$

$$\frac{P_{00}^2}{P_0^2} \geqslant \frac{\sigma_0^2}{a_0^2}\, e^{\frac{a_0^2}{\sigma_0^2}-1} \quad \text{at} \quad a_0^2/\sigma_0^2 \geqslant 1. \tag{40a}$$

It follows from inequality (40) that with small values of $a_0^2/\sigma_0^2$ the ratio $P_{00}^2/P_0^2$ is close to unity, i.e., that the changeover to criterion $P^2$ does not give a noticeable rise in $P^2$. Inequality (40a) shows that with large values of ratio $a_0^2/\sigma_0^2$ the changeover to the $P^2$ criterion results in a substantial increase of functional $P^2$ (or P).

The above evaluations of ratio $P_{00}^2/P_0^2$ and the formulas (34) and (35) lead to the following conclusions with respect to this example. With fixed finite values of $\kappa_e$ and $\gamma$ and with $\kappa \to \infty$ ratio $a_0^2/\sigma_0^2$ tends to zero, hence, under these conditions a changeover to the $P^2$ criterion is not expedient, since it does not produce a marked gain in $P^2$. With the fixing of the value of $\gamma^2$ and $\gamma_e \to \infty$ and when $\kappa \to 0$ the ratio $P_{00}^2/P_0^2$ tends to $\infty$, and hence with large $\kappa_e$ and small $\kappa$ a changeover to the $P^2$ criterion is expedient, since it ensures a substantial gain in $P^2$ (or P).

Example 2. The conditions of the problem are the same but with the difference that now the value of $\underline{v}$ is not a given quantity, but a random one distributed according to the law of equal probability in the range $-v_1$ to $+v_1$. The random quantity $\underline{v}$ and random function $\underline{z}$ are not interrelated, i.e., $M[v\,z] = 0$. In this example the probability that $|\Delta| < \epsilon$, where $\epsilon$ is small, is calculated from formula

$$P_\epsilon = P_\epsilon(|\Delta| < \epsilon) = \frac{2\epsilon}{\sqrt{2\pi}\sqrt{3}\sigma_2}\int_0^{\sqrt{3}\frac{\sigma_2}{\sigma_1}} e^{-\frac{x^2}{2}}\,dx,$$

where $\sigma_1$ is the mean-square deviation of the error component at the system output due to the input error $\underline{z}$, $\sigma_2$ is the mean-square deviation of the error component at the system output due to the random quantity $\underline{v}$. Variance $\sigma_1^2$ is calculated from formula (35) and the variance $\sigma_2^2$ from formula (34). Moreover, quantity $\gamma$ in formulas (34) and (35) is calculated from formula

$$\gamma^2 = \frac{\alpha^2}{\sigma_v^2} = \sqrt{3}\,\frac{\alpha^2}{v_1^2}.$$

Similarly to the first example, we shall use instead of $P_\epsilon$ quantity P which is proportional to it:

$$P = \frac{1}{\sigma_2}\int_0^{\sqrt{3}\frac{\sigma_2}{\sigma_1}} e^{-\frac{x^2}{2}}\,dx, \tag{41}$$

and in addition to P we shall also consider quantity $P^2$.

By taking the same initial data (36) as in the first example and utilizing formulas (34), (35), and (41) it is easy to plot a graph of function $P^2(\theta)$ and $P(\theta)$ in the range of $\theta$ between 0 and 1. These graphs permit one to determine the maximum values of $P^2$ and P and corresponding ratios $P^2(\theta_0)/P^2(1)$ and $P(\theta_0)/P(1)$.

Below are the values of these ratios:

$$\frac{P^2(0.43)}{P^2(1)} = 1{,}18; \quad \frac{P(0.43)}{P(1)} = 1{,}09.$$

Thus, with the initial conditions discussed above the changeover to the P criterion leads to an increase in $P^2$ of 18% and in P of 9%. With other initial conditions the gain in P (or $P^2$) will be greater or smaller. The conclusions drawn with respect to the increment in P or $P^2$ with relation to parameters $\kappa$ and $\kappa_e$ deduced at the end of the first example hold for this example as well.

## SUMMARY

The paper gives the necessary and sufficient conditions for an extreme in functional $I = \Phi\left\{I_I[k(t)], \ldots, I_{n+1}[k(t)]\right\}$ for the case when the necessary and sufficient conditions for the extreme in the functional $I_I = \theta_1 I_1 + \ldots + \theta_n I_n + I_{n+1}$ are known.

The results obtained in the paper can be used for selecting optimum dynamic systems.

Particular examples at the end of the paper show that the use of a more complex criterion (P or $P^2$) leads in certain cases to an increase in the quality of the selected system (i.e., P or $P^2$).

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Received December 9, 1957

* See English translation.

# MATHEMATICAL SIMULATION OF DRY FRICTION*

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Simulation of dry friction is considered in three cases: when starting friction is equal to kinetic friction, when it exceeds the latter, and when it depends on the duration of the contact at rest. Tried out circuits for simulating the movements of solid bodies with one degree of freedom and simulating minor oscillations of a gimbal gyroscope axis are described. A method of simulating, by means of differentiating amplifiers, systems whose differential equations have not been solved with respect to the higher order derivatives is given.

Dynamic systems often contain mechanical devices. In automatic control systems, for instance, mechanical devices are usually employed in sensing elements and final control stages. In view of this it often becomes necessary to simulate nonlinear processes due to dry friction.

It is usual to base mathematical simulation on the analysis of dry friction given by the French scientist Amonton (1663-1705) [1]. It is assumed that at rest the friction force balances out the effort tangential to the sliding surface. Movement will result when the above external effort exceeds the maximum possible value of the friction force (the starting friction force) which is equal to the product of the friction coefficient and the force normal to the sliding surface. During sliding the friction force is normally taken to be in the opposite direction to the movement, constant and equal to the starting friction force. Sometimes, the kinetic friction force is taken, according to Euler, to be smaller than the starting force.

This paper deals with the problem of mathematical simulation of dry friction whose verbal description is given above.

It is usual to base the mathematical simulation of dry friction on the formula expressing the friction force in terms of the sliding speed by means of Kronecker's formula

$$T = -kP \operatorname{sign} V, \tag{1}$$

where T is the friction force, $\underline{k}$ is the coefficient of friction, P is the normal force and V is the speed of sliding

$$\operatorname{sign} V = \begin{cases} 1 \text{ with } V > 0, \\ -1 \text{ with } V < 0. \end{cases}$$

Expression (1) does not represent the behavior of the friction force when the sliding contact surfaces are at rest. In a number of cases, however, when the starting friction force is equal to the kinetic, one expression (1) can be used both for the mathematical representation of the dry friction processes and for simulation. By using expression (1) it becomes possible formally to represent the forward rectilinear motion of the body by equation

* Read at the seminar on the theory of automatic control at the Institute of Automation and Remote Control of the USSR, Academy of Sciences.

$$m\ddot{x} = F - kP \operatorname{sign} \dot{x}, \tag{2}$$

where $m$ is the mass of the body, $x$ is the coordinate determining the body's position, F is the external force acting along the friction surface.

If $|F| < kP$, $\dot{x}(0) = 0$, the solution $\dot{x} \equiv 0$ becomes a definite switching state of sliding [2] simulated by small high-frequency parasitic oscillations around the zero value of quantity $x$. This produces an effect similar to the effect of vibrational linearization; and the quantity $x$ varies with a mean speed proportional to F.

The additional verbal definition of function (1) as a function of two variables—the speed of sliding and the external force—is not used directly in simulating, but the discontinuous function of speed (1) is approximated by a continuous one with a steeply sloping section. In this case it is possible to consider that dry friction at rest changes to large viscous friction. In order to simulate the excess of the starting friction force over the kinetic one, the approximating function of speed is varied so that its extremes come at the boundaries of the stated steeply-sloping section. In this case, as in the preceding one, the presence of a sloping section of the characteristic gives rise to parasitic deviations in the simulation process.

The number of variations in simulating dry friction can be increased not only as the result of a more accurate interpretation of the dry friction law, by taking into consideration such circumstances as, for instance, the relation between the friction force and the duration of the state of rest contact, but also by examining various dynamic systems with friction for properties which could be eliminated. The latter method leads to relationships which cannot be obtained directly from simplifying equations of the type of [2] by neglecting certain terms.

Let us now examine the forward rectilinear movement of a solid body with dry and viscous friction and a force of inertia. By using expression (1) let us write the motion equation in the form:

$$m\dot{x} = F - kP \operatorname{sign} \dot{x} - l\dot{x} - nx, \tag{3}$$

where $l$ is the coefficient of viscous friction, $n$ is the coefficient of the force of inertia.

The representation of the static relation between $x$ and F, which is attained at sufficiently slow variations of F, by means of omitting the dynamic terms $m\dot{x}$ and $l\dot{x}$ in equation (3), as it is done in [3, 4] leads to the relationship

$$nx = F - kP \operatorname{sign} \dot{x}, \tag{4}$$

which does not represent the actual continuous relation between $x$ and F, prevailing in this case. In the above mentioned papers formula (4) is corrected to a certain extent by additional verbal definitions.

Let us also examine the forward rectilinear movement of a body with dry and viscous friction, but with negligible inertia. The relationship between the speed of the body and the external force for this case is shown in Fig. 1, a. If at the same time the excess of the starting friction force over the kinetic friction force is taken into account, the relationship shown in Fig. 1, b will be obtained, where the transition from the horizontal to the sloped section takes place at points $F_1$ and $-F_1$ and the reverse action at points $F_2$ and $-F_2$ [5].

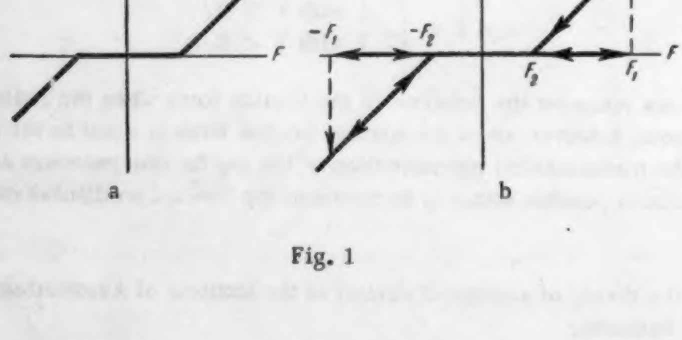


Fig. 1

Let us now describe the results obtained by the author in simulating dry friction on model MN-8.

## The Case of the Starting Friction Force Being Equal to the Kinetic Friction Force

As it has already been stated, the characteristic shown in Fig. 1, a represents the relations between the speed of the external force and the forward rectilinear movement of a solid body with dry and viscous friction and negligible inertia. This characteristic is produced with an opposite sign by the dead zone unit (DZU) of model MN-8. The mathematical representation of a more general case (taking into account the inertia of the body) can be obtained by adding to the external force the force of inertia. In simulating it is possible to use for this purpose the differentiating unit (DU) which differentiates and reverses the sign.

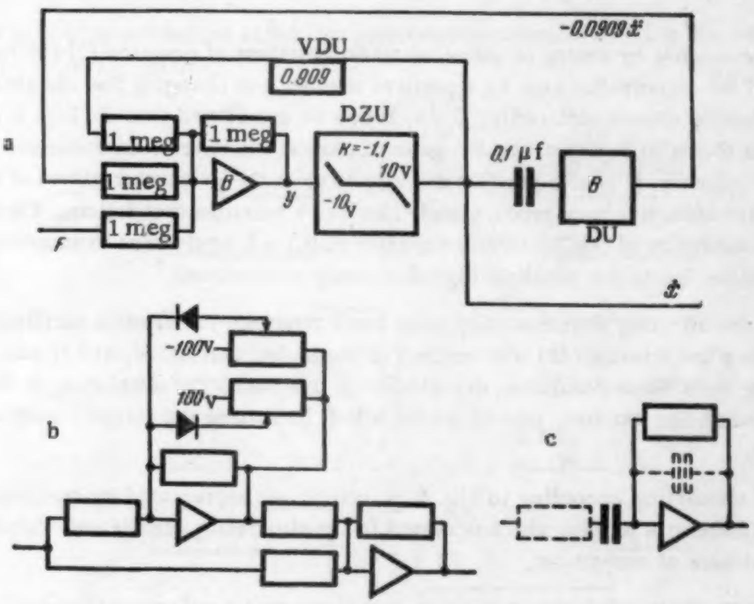


Fig. 2

Moreover, if a movement with dry, but without viscous friction has to be represented, the viscous friction force with a reversed sign can be added to the external force. The circuit simulating the movement of the body in this case is represented in Fig. 2, a, where the block with the inscription VDU stands for the voltage dividing unit. The schematic of the DZU when used for reproducing the characteristic of Fig. 1, a * is given in Fig. 2, b. In Fig. 2, a the DZU is shown adjusted to remain insensitive up to 10 v and k the tangent of the slope angle of the external sections of its characteristic, is made equal to 1.1. The schematic of the block DU, which belongs to the auxiliary part of model MN-8, is shown in Fig. 2, c. The circuit contains small parasitic input resistances and a feedback capacitance, shown by a dotted line. When simulating friction, the input voltage to the differentiating unit was impressed through an additional capacitor of 0.1 μf, whose terminals were connected to the patch bay. With the additional capacity in circuit, the unit differentiated and multiplied by -0.0909 with parasitic time constants of 0.0024 sec and 0.0005 sec. The additional capacity was introduced to eliminate high-frequency oscillations. For the same purpose, differentiation and summation was carried out on high-frequency amplifiers model MN-8 with a band-pass of 1000 cps. The high-frequency amplifiers are marked on the circuit by the letter B.

It is easy to check that the equation for the characteristic of Fig. 1, a has the form:

$$c_1\dot{x} = (F - c_2)\,\frac{1 + \operatorname{sign}(F - c_2)}{2} + (F + c_2)\,\frac{1 - \operatorname{sign}(F + c_2)}{2}. \tag{5}$$

---

* By means of the DZU of model MN-8 it is also possible to represent a characteristic with a limitation of coordinates.

By adding to the external force the force of inertia and subtracting the viscous friction force we obtain the equation:

$$c_1\dot{x} = (F - c_3\ddot{x} + c_1\dot{x} - c_2)\,\frac{1 + \operatorname{sign}(F - c_3\ddot{x} + c_1\dot{x} - c_2)}{2} +$$
$$+ (F - c_3\ddot{x} + c_1\dot{x} + c_2)\,\frac{1 - \operatorname{sign}(F - c_3\ddot{x} + c_1\dot{x} + c_2)}{2}. \qquad (6)$$

By substituting the coefficients with their numerical values according to Fig. 2, a we obtain

$$\dot{x} = (1{,}1\;F - 0{,}1\ddot{x} + \dot{x} - 11)\,\frac{1 + \operatorname{sign}(1{,}1F - 0{,}1\ddot{x} + \dot{x} - 11)}{2} +$$
$$+ (1{,}1F - 0{,}1\ddot{x} + \dot{x} + 11)\,\frac{1 - \operatorname{sign}(1{,}1F - 0{,}1\ddot{x} + \dot{x} + 11)}{2}. \qquad (7)$$

It should be noted that by means of identical transformations of equation (7) (by multiplying its parts and the argument of the signum-function by a positive number and changing the variables) the absolute values of the coefficients can be chosen arbitrarily. Thus, it can be considered that the Fig. 2, a circuit, with the values of parameters shown in it, represents the general case of a forward rectilinear movement of a solid body with dry friction. Obviously, it is also possible directly to reset the parameter values of the simulating circuit. In some instances, however, this may produce high-frequency parasitic oscillations. Oscillations are caused, for instance, by the exclusion of the additional capacity of 0.1 $\mu$f, equivalent to increasing the gain of the differentiating amplifier (up to the parasitic high-frequency oscillations).

A simulating circuit using differentiating units has a tendency to parasitic oscillation. It was not found possible, however, to solve equation (6) with respect to the higher derivative, and it was therefore necessary to use differentiating units when simulating dry friction in this case. For simulating in the differential form facilities are provided in the auxiliary part of model MN-8 for setting the initial conditions on the differentiating amplifiers.

The results of simulating according to Fig. 2, a circuit are represented by oscillograms. * The oscillogram of Fig. 3, a represents a process which occurred in the simulating circuit and which corresponds to the following physical picture of movement.

At first the external force F is equal to zero; next it assumes a value equal to half the starting friction force but the body remains at rest. Then force F suddenly jumps to a value one and a half times greater than the starting friction force, and sliding begins. The change in the sign of the external force leads to rapid braking and a change in the direction of sliding. When the external force becomes smaller than the kinetic friction force, the speed decreases and relative stability is restored.

The only difference between the oscillograms shown in Figs. 3, b and 3, c and the one just described lies in the fact that in Fig. 3, b the VDU was not set to a gain of 0.909 but to 0.959 and in Fig. 3, c to 0.859. Also in the first case, the feedback coefficient of the DZU was greater than one, and in the second case smaller than one, which corresponds to introducing viscous friction with different signs. There is no distortion of the simulated processes in this case either.

Above circuit can be used as part of another circuit for simulating more complex processes. Let us examine, as an example, the oscillations of an axis of a gyroscope freely suspended on a gimbal in a position nearly perpendicular between the suspension axis and the gyroscope rotor, with dry friction present in the bearings.

Let us introduce the following notation: $M_1$ is the external moment about the axis of the gimbal internal suspension ring, $y_1$ is the angular velocity of the gyroscope rotor axis round the internal suspension-ring axis, $I_1$ the moment of inertia of the mass involved in movement round the internal suspension-ring axis, $c_1$ the moment of starting friction round the internal suspension-ring axis, $M_2$, $y_2$, $I_2$, and $c_2$ are similar quantities for the movement round the external suspension-ring axis, and H is the kinetic moment of the gyroscope.

* For the sake of a more convenient presentation the parts of the same curve at the points of discontinuity are joined by vertical lines.

Let us now write the equations of the moments about both axes of the gyroscope gimbal suspensions [6] by using the form of expression for dry friction, already obtained:

$$
\begin{aligned}
c_3 y_1 = {} & (M_1 - I_1\ddot{y}_1 - Hy_2 + c_3 y_1 + c_1)\,\frac{1 - \operatorname{sign}(M_1 - I_1\ddot{y}_1 - Hy_2 + c_3 y_1 + c_1)}{2} + \\
& + (M_1 - I_1\ddot{y}_1 - Hy_2 + c_3 y_1 - c_1)\,\frac{1 + \operatorname{sign}(M_1 - I_1\ddot{y}_1 - Hy_2 + c_3 y_1 - c_1)}{2}, \\
c_4 y_2 = {} & (M_2 - I_2\ddot{y}_2 + Hy_1 + c_4 y_2 + c_2)\,\frac{1 - \operatorname{sign}(M_2 - I_2\ddot{y}_2 + Hy_1 + c_4 y_2 + c_2)}{2} + \\
& + (M_2 - I_2\ddot{y}_2 + Hy_1 + c_4 y_2 - c_2)\,\frac{1 + \operatorname{sign}(M_2 - I_2\ddot{y}_2 + Hy_1 + c_4 y_2 - c_2)}{2}.
\end{aligned}
\tag{8}
$$

Let us take the following numerical values for gyroscope parameters $I_1 = 5$ g cm sec$^2$, $I_2 = 20$ g cm sec$^2$, $c_1 = c_2 = 0.5$ g cm, $H = 18{,}000$ g cm sec.

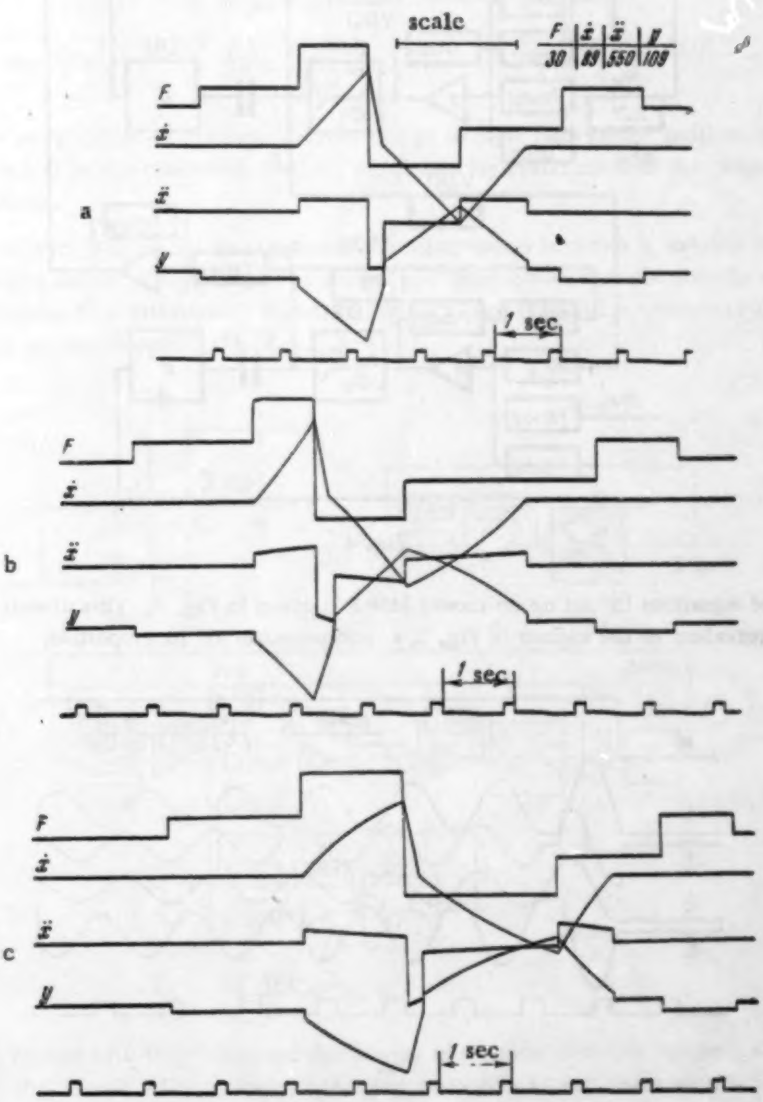


Fig. 3

By giving their numerical values to the coefficients in equation (8) and using substitutions $t = 300^{-1}\,\tau$, $y_1 = 3 \cdot 10^{-6} z_1$, $y_2 = 0.75 \cdot 10^{-6} z_2$, we obtain

$$
\begin{aligned}
z_1 &= (20M_1 - 0{,}09\overset{*}{z}_1 - 0{,}27z_2 + z_1 + 10)\,\frac{1 - \operatorname{sign}(20M_1 - 0.09\overset{*}{z}_1 - 0.27z_2 + z_1 + 10)}{2} + \\
&\quad + (20M_1 - 0{,}09\overset{*}{z}_1 - 0{,}27z_2 + z_1 - 10)\,\frac{1 + \operatorname{sign}(20M_1 - 0.09z_1 - 0.27z_2 + z_1 - 10)}{2}, \\
z_2 &= (20M_2 - 0{,}09\overset{*}{z}_2 + 1{,}1z_1 + z_2 + 10)\,\frac{1 - \operatorname{sign}(20M_2 - 0.09\overset{*}{z}_2 + 1.1z_1 + z_2 + 10)}{2} + \\
&\quad + (20M_2 - 0{,}09\overset{*}{z}_2 + 1{,}1z_1 + z_2 - 10)\,\frac{1 + \operatorname{sign}(20M_2 - 0.09\overset{*}{z}_2 + 1.1z_1 + z_2 - 10)}{2}.
\end{aligned}
\tag{9}
$$

**Where the asterisk means differentiation with respect to $\tau$.**

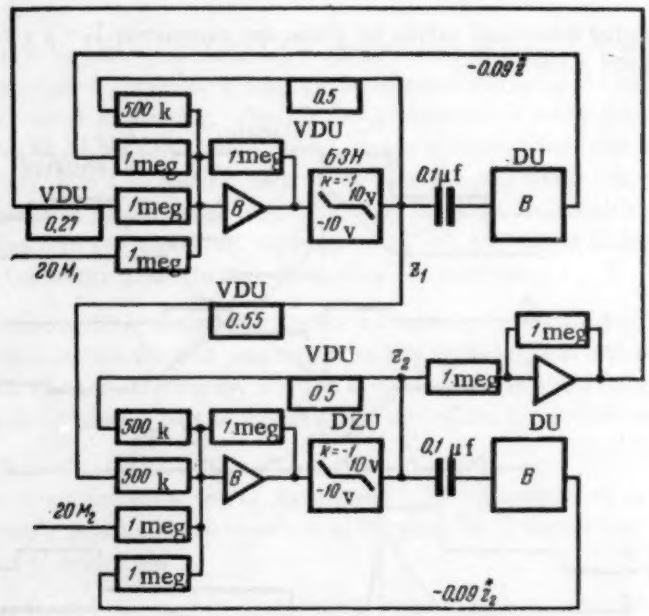


Fig. 4

**The circuit of equations (9) set up on model MN-8 is given in Fig. 4. This circuit consists of two parts, each of which is equivalent to the circuit of Fig. 2, a and possesses all its properties.**

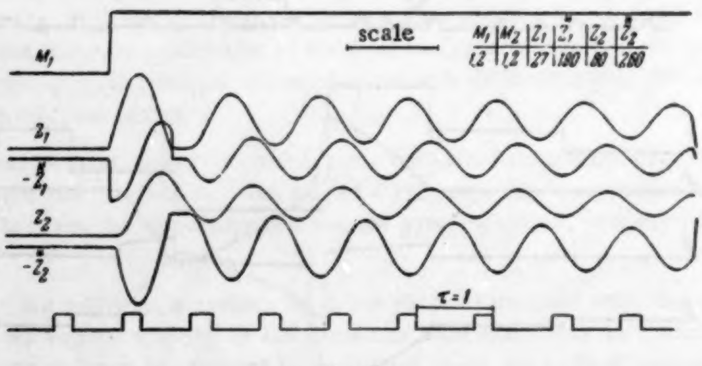


Fig. 5

**The oscillogram of Fig. 5 shows the process of simulating the gyroscope movement about the internal axis of the gimbal suspension, due to a unit step action of $M_1$.**

It should be noted that in various actual gyroscopic instruments backlash in bearings and unbalanced suspension can considerably affect the gyroscope axis movement.

## The Case of the Starting Friction Force Being Greater Than The Kinetic Friction Force

Let us now consider a more complicated case of dry friction when the rise of the starting friction force above the kinetic friction force is taken into account. We shall examine among the various simulating circuits not the simplest, which are attained with the help of so-called signature units of model MN-8, but those which are least subject to parasitic high-frequency oscillations.

They can be examined in the same way as before if instead of the characteristic of Fig. 1, a that of Fig. 1, b is used, which can be represented formally as follows:

$$\begin{aligned} c_1\dot{x} &= (F + y_1 + y_2 - c_2)\,\frac{1+\mathrm{sign}\,(F+y_1+y_2-c_2)}{2} + \\ &+ (F + y_1 + y_2 + c_2)\,\frac{1-\mathrm{sign}\,(F+y_1+y_2+c_2)}{2}, \\ y_1 &= c_3\,\frac{1+\mathrm{sign}\,(F+y_1-c_2)}{2}, \quad y_2 = -c_3\,\frac{1-\mathrm{sign}\,(F+y_2+c_2)}{2}. \end{aligned} \tag{10}$$

We shall ascribe to functions of the type $y = \mathrm{sign}\,(x + y)$ in their two-valued portion, both now and henceforth, the value equal to the preceding portion, which usually corresponds to the properties of the circuits embodying these functions.

The first of the system (10) equations represents the relationship between $\dot{x}$ and the sum $F + y_1 + y_2$ similar to the relationship shown in Fig. 1, a. The second and third equations establish the relationship between the sum $y_1 + y_2$ and quantity F, a relationship shown in Fig. 6, a and realized by means of DZ units of model MN-8 according to the method shown in Fig. 6, b.

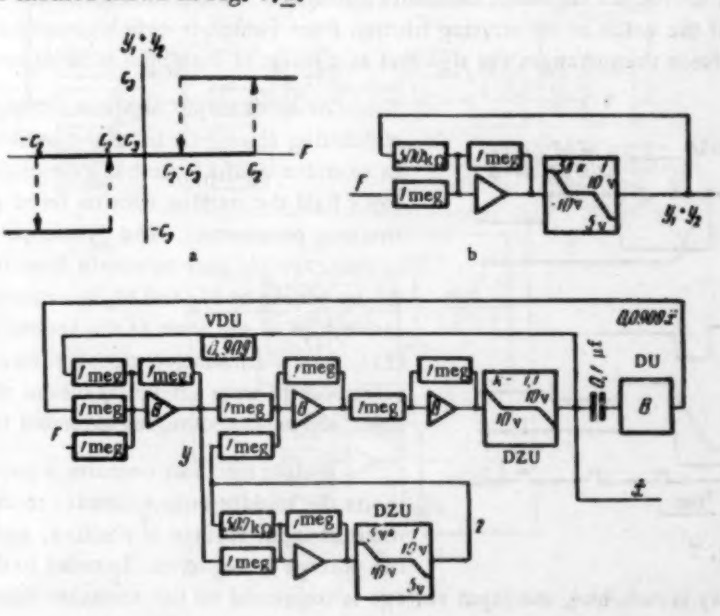


Fig. 6

The simulating circuit of a body's forward rectilinear movement with dry friction, when the starting friction force exceeds the kinetic friction force, operating according to the characteristics of Fig. 1, b is shown in Fig. 6, c.

The movement equations in this case will be

$$c_1\dot{x} = (F - c_4\ddot{x} + c_1\dot{x} - c_2 + y_1 + y_2)\,\frac{1 + \operatorname{sign}(F - c_4\ddot{x} + c_1\dot{x} - c_2 + y_1 + y_2)}{2}$$

$$+ (F - c_4\ddot{x} + c_1\dot{x} + c_2 + y_1 + y_2)\,\frac{1 - \operatorname{sign}(F - c_4\ddot{x} + c_1\dot{x} + c_2 + y_1 + y_2)}{2}, \qquad (11)$$

$$y_1 = c_3\,\frac{1 + \operatorname{sign}(F - c_4\ddot{x} + c_1\dot{x} + y_1 - c_2)}{2},$$

$$y_2 = -\,c_3\,\frac{1 - \operatorname{sign}(F - c_4\ddot{x} + c_1\dot{x} + y_2 + c_2)}{2}.$$

By giving the coefficients their numerical values, which determine the parameters of Fig. 6,c circuit, we obtain

$$\dot{x} = (1{,}1F - 0{,}1\ddot{x} + \dot{x} - 11 + 1{,}1y_1 + 1{,}1y_2)\times$$

$$\times\frac{1 + \operatorname{sign}(1{,}1F - 0{,}1\ddot{x} + \dot{x} - 11 + 1{,}1y_1 + 1{,}1y_2)}{2} + (1{,}1F - 0{,}1\ddot{x} + \dot{x} + 11 + 1{,}1y_1 + 1{,}1y_2)\times$$

$$\times\frac{1 - \operatorname{sign}(1.1F - 0{,}1\ddot{x} + \dot{x} + 11 + 1.1y_1 + 1{,}1y_2)}{2},$$

$$y_1 = 5\,\frac{1 + \operatorname{sign}(1.1F - 0{,}1\ddot{x} + \dot{x} + 1{,}1y_1 - 11)}{2}, \qquad (12)$$

$$y_2 = -\,5\,\frac{1 - \operatorname{sign}(1.1F - 0{,}1\ddot{x} + \dot{x} + 1{,}1y_2 + 11)}{2}.$$

The oscillogram in Fig. 7 shows a simulating process in which the external force first changes from zero to 70% of the starting friction force. Next, due to a momentary rise of the external force above the starting friction force, sliding occurs which continues during the subsequent decrease of the external force until it reaches 70% of the value of the starting friction force (which is 40% higher than the kinetic friction force). The external force then changes the sign and as a result of braking a state of rest is restored.

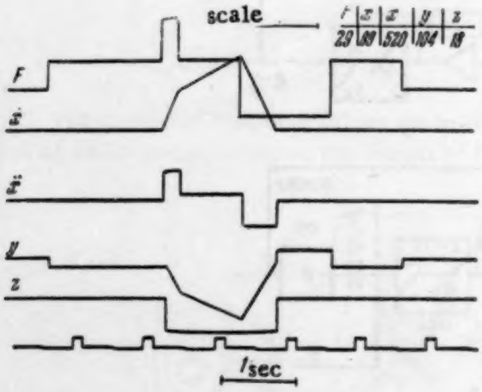


Fig. 7

As an example of a more complex process whose simulating circuit includes the one described above, let us examine again a gimbal gyroscope with a kinetic friction force half the starting friction force and with the same remaining parameters. The gyroscope movement equations in this case are easy to obtain from the gyroscope equation (8) by adding to $M_1$ and $M_2$ quantities determined by relationships of the form of the second and third equations (11). In the simulation circuit shown in Fig. 8, the latter relationships were set-up by means of model MN-8 signature units, shown as rectangles extended in the vertical direction.

A signature unit contains a polarized relay, which shorts the middle output contact to the bottom one when the unit input voltage is positive, and to the top one when the voltage is negative. In order to decrease the parasitic processes when the relay is switched, the input voltage is impressed on the armature through an electronic trigger included in the unit. The input and the three output terminals of the signature unit are placed in the patch bay of the model. The input signals for the signature units are shaped by specially provided high gain amplifiers (for the purpose of decreasing the effect of the above mentioned parasitic processes due to relay switching). The feedback circuits of these amplifiers include diode limiters, not shown in the circuit, which cut-off the output voltage at 100 v thus keeping the amplifiers within the permissible limits of operation.

The author intends to devote a separate paper to the use of signature units.

The oscillogram of Fig. 9, a shows the simulation process of gyroscope oscillations due to an external moment $M_1$, about the internal suspension ring axis, a moment which is greater than the starting friction moment by 10%. The oscillogram of Fig. 9, b shows a process in which at first the external moments are equal to zero, then moment $M_2$ of 0.3 g cm (the starting friction is 0.5 g cm) is applied about the external suspension ring axis. The system remains at rest. Next $M_2$ assumes the value of 0.8 g cm and when movement begins $M_2$ falls again to 0.3 g cm and the system reverts to a state of rest. With a repeated rise in $M_2$ to 0.8 g cm the subsequent fall to 0.3 g cm occurs with a different type of gyroscope movement. Moreover, oscillations are produced at a lower frequency than in the case of the oscillogram of Fig. 9, a.

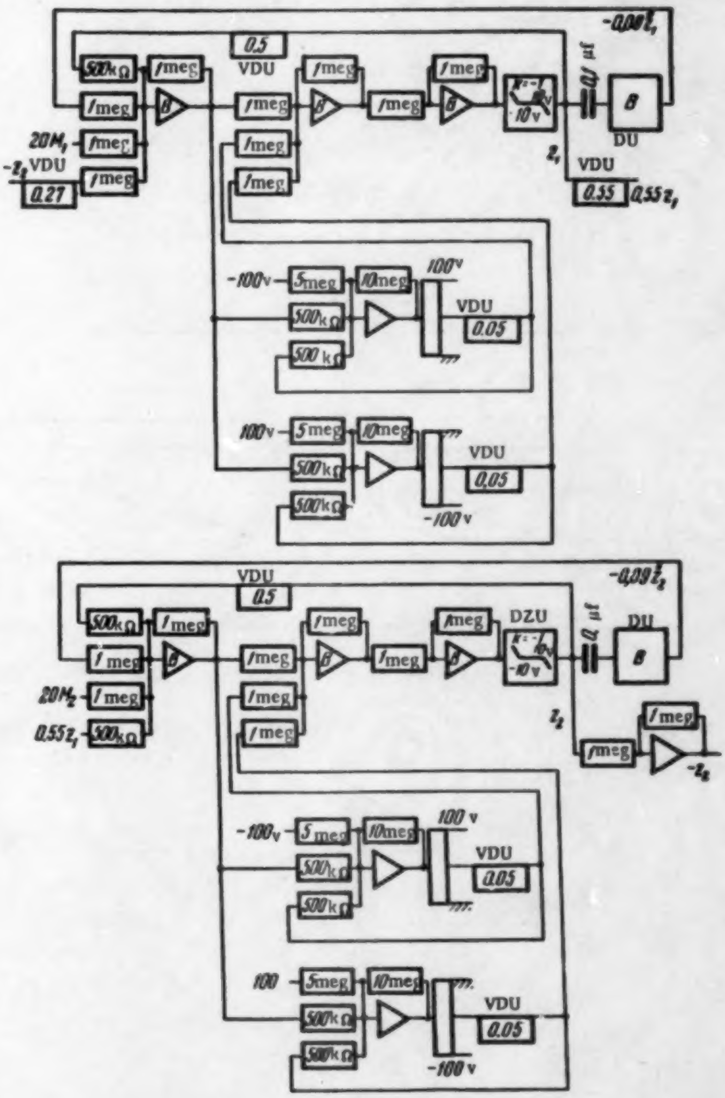


Fig. 8

## The Case of the Starting Friction Force Depending on the Duration of the Contact At Rest

Let us now assume as it was done in [7 and 8] that the excess of the starting friction force over the kinetic friction force is a function of the time the contact between the rubbing surfaces remains at rest and that it is expressed by formula

$$T(\tau) = T_{\infty} - (T_{\infty} - T_0)\, e^{-\varkappa\tau}, \tag{13}$$

where $T_\infty$ is the friction force when the contact remains an infinitely long time at rest, $T_0$ is the zero contact resting-time friction-force, equal to the sliding friction force, and $\tau$ is the duration of the contact resting time.

As before we shall base our reasoning on a relationship of the type of (5) whose parameter $c_2$ can be increased in relation to the contact resting time $\tau$.

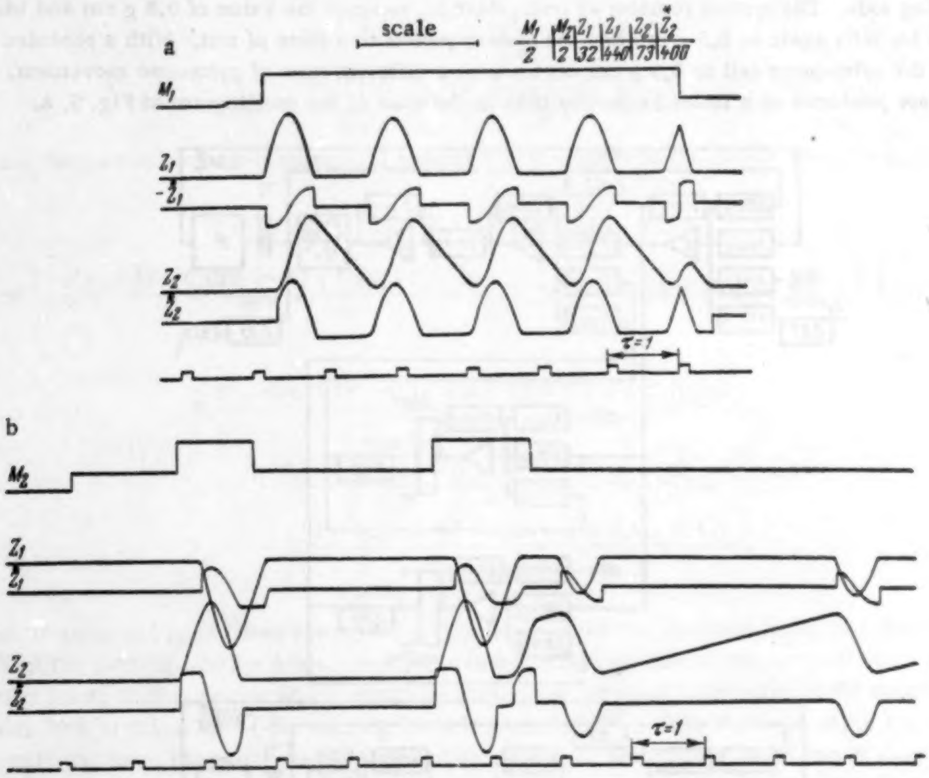


Fig. 9

When examining the forward rectilinear movement of a body with dry and viscous friction and insignificant inertia, let us use the approximate relationships

$$
\begin{aligned}
c_1\dot{x} &= (F - c_2 - y_1)\frac{1+\operatorname{sign}(F-c_2-y_1)}{2} + (F + c_2 + y_1)\frac{1-\operatorname{sign}(F+c_2+y_1)}{2}, \\
y_1 &= y_2\frac{1-\operatorname{sign}(F-c_2-y_1)}{2}\,\frac{1+\operatorname{sign}(F+c_2+y_1)}{2}, \\
\dot{y}_2 + y_2\Big\{c_3 + c_4&\Big[\frac{1+\operatorname{sign}(F-c_2-y_1)}{2} + \frac{1-\operatorname{sign}(F+c_2+y_1)}{2}\Big]\Big\} = \\
&= c_5\frac{1+\operatorname{sign}(F+c_2+y_1)}{2}\,\frac{1-\operatorname{sign}(F-c_2-y_1)}{2}.
\end{aligned} \tag{14}
$$

These equations represent the process with increasing accuracy as $c_4$ increases. It is easy to see that the right hand side of the third equation (14) cannot be negative. With a positive initial value of $y_2$ its subsequent values will also be positive. Quantity $y_1$ which, according to the second equation, is equal either to zero or to $y_2$ will not be negative either. Thus the value of the friction force, determined by the first equation (14), has a minimum equal to $c_2$ and can rise on account of the $y_1$ term.

At first let $|F| < c_2 + y_1$ expressing a stationary contact. Equation (14) will then take the form:

$$
\begin{aligned}
c_1\dot{x} &= (F - c_2 - y_1)\,\frac{1+\operatorname{sign}(F-c_2-y_1)}{2} + (F + c_2 + y_1)\frac{1-\operatorname{sign}(F+c_2+y_1)}{2}, \\
y_1 &= y_2, \quad \dot{y}_2 + c_3y_2 = c_5.
\end{aligned} \tag{15}
$$

Equation (15) represents the rise in the starting friction according to relationship (13). Now as a result of external action let $|F| > c_2 + y_1$. In this case equations (14) become equivalent to equations

$$
\begin{gathered}
c_1\dot{x} = (F - c_2 - y_1)\,\frac{1 + \mathrm{sign}\,(F - c_2 - y_1)}{2} + (F + c_2 + y_1)\,\frac{1 - \mathrm{sign}\,(F + c_2 + y_1)}{2}\,, \\
y_1 = 0,\ \dot{y}_2 + (c_3 + c_4)\,y_2 = 0,
\end{gathered}
\tag{16}
$$

Equations (16) represent sliding with friction $c_2$. Moreover, the larger the sum $c_3 + c_4$, the quicker will the intermediate variable $y_2$ tend to zero. If it happens further that, as a result of an external action, $|F|$ becomes smaller than $c_2$, the process will then be represented by Equation (15).

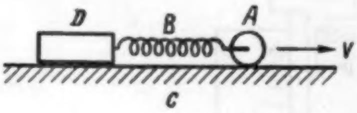


Fig. 10

Let us note that from equations (14) it is possible to pass to less accurate but at the same time simpler equations, for instance, by substituting $y_1$ for $y_2$ in the first and third equations and by omitting the second equation. In addition, the right-hand side of the third equation can be substituted by $c_5$, providing the value of $c_4$ is sufficiently large.

Figure 10 shows a device in which cylinder A, moving with a constant velocity V, draws by means of spring B a body D over surface C. Under the influence of the rise of the starting friction force above the kinetic friction force there arises an intermittent movement of the body.

In simulating above case of dry friction, equation (14) was used, somewhat simplified in the manner indicated above at the expense of the right-hand side of the third equation. By adding to the external force the force of inertia and subtracting the viscous friction force, and by assuming the external force to be determined according to Fig. 10 we obtain equation

$$
\begin{gathered}
c_1\dot{x} = (F - c_6\ddot{x} + c_1\dot{x} - c_2 - y_1)\,\frac{1 + \mathrm{sign}\,(F - c_6\ddot{x} + c_1\dot{x} - c_2 - y_1)}{2} + \\
+ (F - c_6\ddot{x} + c_1\dot{x} + c_2 + y_1)\,\frac{1 - \mathrm{sign}\,(F - c_6\ddot{x} + c_1\dot{x} + c_2 + y_1)}{2}\,, \\
y_1 = y_2\,\frac{1 - \mathrm{sign}\,(F - c_6\ddot{x} + c_1\dot{x} - c_2 - y_1)}{2} \cdot \frac{1 + \mathrm{sign}\,(F - c_6\ddot{x} + c_1\dot{x} + c_2 + y_1)}{2}\,, \\
\dot{y}_2 + y_2\left\{c_3 + c_4\left[\frac{1 + \mathrm{sign}\,(F - c_6\ddot{x} + c_1\dot{x} - c_2 - y_1)}{2} + \right.\right. \\
\left.\left. + \frac{1 - \mathrm{sign}\,(F - c_6\ddot{x} + c_1\dot{x} + c_2 + y_1)}{2}\right]\right\} = c_5, \\
\dot{F} = V - \dot{x}.
\end{gathered}
\tag{17}
$$

The simulation circuit corresponding to equation (17) with $c_1 = 1$, $c_2 = 10$, $c_3 = 0.2$, $c_4 = 500$, $c_5 = 3$, $c_6 = 0.0909$ is shown in Fig. 11. In it, signature units containing two polarized relays, operated by the same input signal, were used. Both in the circuit and the patch bay, the three top output contacts are switched by one, and the three bottom ones by the other relay.

The oscillogram of Fig. 12 represents the process of simulating intermittent motion arising, after a long state of rest, as the result of a unit step action V. It clearly shows the peculiarity of this process consisting in the first speed pulse being larger than the rest. This peculiarity was first explained by A. Iu. Ishlinskii and I. V. Kragel'skii [7 and 8].

It should be noted that in practice, when simulating according to Fig. 11, the drop in the intermediate value of $y_2$ at the beginning of the simulation of sliding can develop slower than it should according to equation (17) since the voltage at the integrator input, shaped by the summing amplifier, is limited by the diodes to the scale of the model, i.e., to 100 v. In this case it did not influence the results of simulation since the duration of the speed pulses in the process under consideration was sufficiently large. The parasitic discharge time of the integrating condenser can be reduced to a minimum by appropriately shorting its plates through

a signature unit. This possibility, however, is considered to be beyond the scope of the present investigations since the patch bay of model MN-8 does not provide for such an operation.

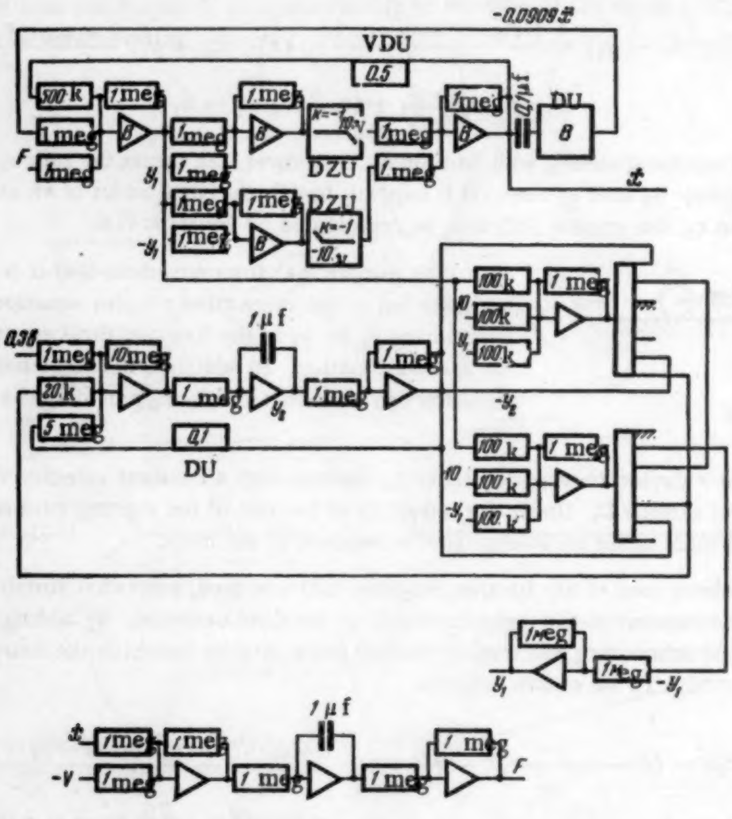

Fig. 11

The above results, obviously, do not exhaust the problem of simulating dry friction. Its examination can be extended. For instance, it is useful to examine the simulation of the forward movement of a body over a plane, or the movement of a shaft tightly seated in a cylindrical bearing with two degrees of freedom, and similar but more complex cases of nonlinear processes involving dry friction.

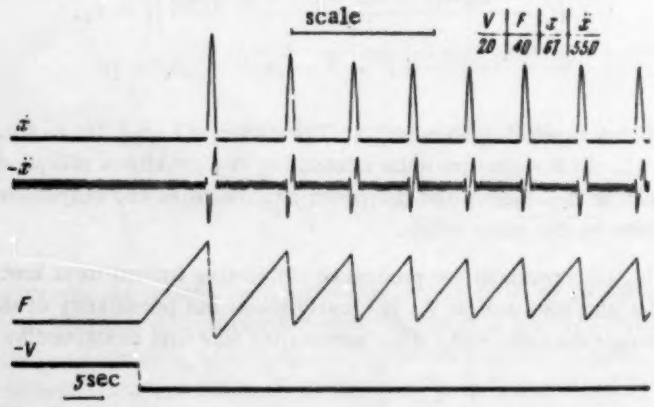

Fig. 12

In the first of the above cases the motion equations can be written as follows:

$$(F_x - c_4\ddot{x} + c_2\dot{x})^2 + (F_y - c_4\ddot{y} + c_2\dot{y})^2 = y_1^2,$$

$$y_2 = y_1 \operatorname{sign} y_1,$$

$$y_3 = (y_2 - c_1)\frac{1 + \operatorname{sign}(y_2 - c_1)}{2},$$

$$c_2\dot{x} = y_3 \frac{F_x - c_4\ddot{x} + c_2\dot{x}}{y_2 + c_3\dfrac{1 - \operatorname{sign}(y_2 - c_3)}{2}}, \tag{18}$$

$$c_2\dot{y} = y_3 \frac{E_y - c_4\ddot{y} + c_2\dot{y}}{y_2 + c_3\dfrac{1 - \operatorname{sign}(y_2 - c_3)}{2}},$$

where $c_1 > c_3 \geq 0$, $F_x$ and $F_y$ are the components of the external force along the axes of Cartesian coordinates, $y_1$, $y_2$, and $y_3$ are intermediate variables, and $\underline{x}$ and $\underline{y}$ are the coordinates of the moving body.

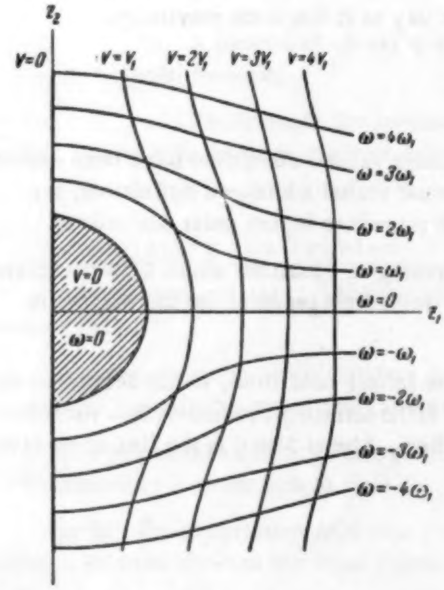


Fig. 13

The starting friction is taken to be equal to the kinetic friction and is represented by $c_1$.

The movement of a shaft, tightly seated in cylindrical bearings, with two degrees of freedom, that of rotation and sliding along the axis, when the difference between the starting friction force and the kinetic friction force is taken into account, can be expressed by the system

$$\left(\frac{F - c_6\ddot{x} + c_4\dot{x}}{c_1}\right)^2 + \left(\frac{M - c_7\ddot{y} + c_5\dot{y}}{c_2}\right)^2 = y_1^2, \tag{19}$$

$$y_2 = y_1 \operatorname{sign} y_1,$$

$$y_3 = \operatorname{sign}\left(y_2 + \frac{1 - c_3}{2} y_3 - \frac{1 + c_3}{2}\right),$$

$$y_4 = (y_2 - c_3)\frac{1 + \operatorname{sign} y_3}{2},$$

$$c_4\dot{x} = y_4 \frac{F - c_6\ddot{x} + c_4\dot{x}}{y_2 + c_8\dfrac{1 - \operatorname{sign}(y_2 - c_8)}{2}},$$

$$c_5\dot{y} = y_4 \frac{M - c_7\ddot{y} + c_5\dot{y}}{y_2 + c_8\dfrac{1 - \operatorname{sign}(y_2 - c_8)}{2}},$$

where $1 > c_3 > c_8 \geq 0$, F is the external force, M is the external moment, $y_1$, $y_2$, $y_3$, and $y_4$ are intermediate variables, $\underline{x}$ is the longitudinal displacement of the shaft, $\underline{y}$ is the shaft's turning angle, $c_1$ is the starting friction force, $c_2$ is the starting friction moment, $c_3$ is the fractional expression for the kinetic friction force in terms of the starting friction force.

The general case of a heavy homogeneous disc sliding along a plane with dry friction (with the starting friction force equal to the kinetic friction force) can be represented by the system

$$\dot{x} = V(z_1, z_2)\frac{F_x - m\ddot{x} + c_1\dot{x}}{z_1},$$

$$\dot{y} = V(z_1, z_2)\frac{F_y - m\ddot{y} + c_1\dot{y}}{z_1}, \tag{20}$$

$$\dot{\alpha} = \omega(z_1, z_2),$$
$$z_1 = \sqrt{(F_x - m\ddot{x} + c_1\dot{x})^2 + (F_y - m\ddot{y} + c_1\dot{y})^2},$$
$$z_2 = M - I\ddot{\alpha} + c_2\dot{\alpha},$$

where functions V and $\omega$ of the two variables $z_1$ and $z_2$ have the form shown in Fig. 13 and the square root is used in its arithmetic meaning. Also where x and y are Cartesian coordinates of the center of the disc, $\alpha$ is the turning angle of the disc, $F_x$ and $F_y$ are the components of the principal vector of the forces acting on the disc, M is the principal moment acting on the disc with respect to its axis of rotation, m is the mass of the disc, I is the polar moment of inertia of the disc.

When functions V and $\omega$ were plotted, in addition to dry friction, viscous friction at the points of contact was also introduced and then the inverse functions $- z_2(V; \omega)$ and $+ z_1(V; \omega)$ were calculated, these functions having the meaning of the principal moment and the modulus of the principal vector of the dry and viscous friction forces with respect to the disc axis. By selecting suitable values for coefficients $c_1$ and $c_2$ the effect of viscous friction on the final result can be eliminated in the same way as it was done previously.

## SUMMARY

1. Differential equations for sliding bodies with dry friction under various conditions have been derived. The equations thus derived are distinguished by the absence of the usual verbal additional definitions, the absence of sliding solutions, and in addition they are not solved with respect to higher order derivatives.

2. Simulation of sliding with dry friction was carried out according to equations which were not solved with respect to higher order derivatives, by means of differentiating amplifiers provided for the purpose in model MN-8.

3. When simulating by means of differentiating amplifiers, the initial conditions, which determine the particular solutions of the equations, are introduced by charging the differentiating condensers in a manner similar to the one used in simulating by means of integrating amplifiers. Model MN-8 is the first of its kind using this arrangement.

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Received December 18, 1957

* See English translation.

# CHOICE OF A POWER UNIT FOR AN OPTIMUM AUTOMATIC CONTROL SYSTEM

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A method of choosing a power unit for an optimum automatic control system is recommended.

To illustrate the recommended method a servosystem whose power unit consists of a dc motor is given as an example.

## 1. Formulating the Problem

A number of articles [1-10] have appeared in recent years dealing with optimum automatic control systems (ACS).

A common posing of the problem is characteristic for all these papers. They take one part of the ACS for granted (usually the power unit) and examine the design of a control system with a minimum response time.

The successful solution of this problem now provides opportunities for a more correct and valid approach to the choice of a power unit as well.

In fact for an optimum ACS with given limitations and a fixed power unit and loading there exists a definite relation between the input signal law of change and the response time.

In this connection it is expedient to know this relation in advance for the most common types of power units and loading in systems working with some typical, for instance, step-by-step disturbance.

If such information is obtained and presented in the form of a family of curves, it will provide the designer of AC systems with answers to a number of important questions.

With their help it will be possible in the first place to determine the response time of an ACS with any given power unit.

If the response time, obtained from the curves, should exceed the permissible value, one can immediately conclude that the preliminary choice of the power unit was not suitable, since even in a system with an optimum control it cannot provide the required speed of operation.

It can happen that with a given load the system will not be able to ensure the required speed of operation even with the best chosen power unit.

The curves will provide an answer to this most important question as well.

It will also be possible to tell from these graphs how near to the optimum is an ACS with a nonoptimum control unit, in order to be able to decide whether it is expedient to change the control unit for the purpose of improving its time characteristics.

The technique of plotting such curves is shown by an example given below.

The design of a servosystem with optimum operation is also examined, its time characteristics, obtained experimentally, are compared with those obtained from the graph.

A simple relay type servosystem powered by a dc motor with separate excitation is given as an example. The motor operates a platform with a constant moment of inertia.

The restricted value of the system is the voltage at the motor windings.

## 2. Motion Equation of an Optimum ACS

Figure 1 gives the schematic of a servosystem.

Motor M, with a separate excitation, operates platform Pl.

The control unit Ctl calculates the best control law for the motor and changes the voltage polarity at the armature windings by means of relay P contacts.

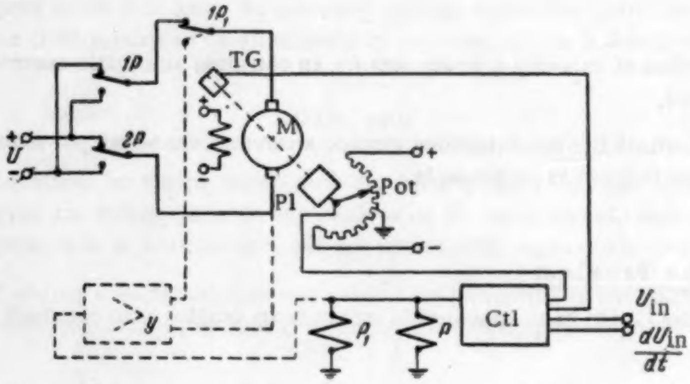


Fig. 1

The platform movement is proportional to voltage $U_{in}$, which is impressed together with its derivative $dU_{in}/dt$ on the control unit input. The voltage from potentiometer Pot slide ($U^*_{in}$) and tachometer generator TG ($dU^*_{in}/dt$), proportional to the platform deviation angle and the platform speed, respectively, are also impressed on the input.

A second relay $P_1$ can be provided at the output of the control unit for connecting the motor armature winding, after balancing, to the linear amplifier, or if required for disconnecting the winding.

The transient process in the motor can be represented by the following equations:

$$U = iR + L\frac{di}{dt} + c_1\omega, \tag{1}$$

$$M_M = c_2 i, \tag{2}$$

$$M_M = J\frac{d\omega}{dt}, \tag{3}$$

where U is the restricted in value direct voltage at the armature winding, $i$ is the armature current, R is the armature resistance, $c_1$ is the back emf coefficient, $\omega$ is the angular velocity, $c_2$ is a constant coefficient, J is the moment of inertia of the platform and motor, referred to the motor shaft, L is the armature inductance, and $M_M$ is the motor turning moment.

In writing these equations certain factors were not taken into consideration, such as the friction force in the gear train, armature reaction, eddy currents and hysteresis in the motor.

The armature self-induction coefficient was assumed constant.

Experimental data, given below, shows that for the case under consideration the omission of above quantities did not make any appreciable difference to the duration of the transient process.

Using (1) - (3) let us write the velocity differential equation in the form:

$$\frac{d^2\omega}{dt^2} + \frac{R}{L}\frac{d\omega}{dt} + \frac{c_1c_2}{JL}\omega = \frac{c_2}{JL}U \tag{4}$$

or

$$\frac{d^2\omega}{dt^2} + \frac{1}{T_e}\frac{d\omega}{dt} + \frac{1}{T_e T_{em}}\omega = \frac{1}{T_e T_{em}}\frac{U}{c_1}, \tag{4'}$$

where $T_e = \frac{L}{R}$ is the electrical time constant, $T_{em} = \frac{JR}{c_1c_2}$ the electromechanical time constant.

The velocity equation for dynamic braking by reversing the armature will differ from (4) only by the sign of the right-hand side of the equation:

$$\frac{d^2\omega}{dt^2} + \frac{1}{T_e}\frac{d\omega}{dt} + \frac{1}{T_e T_{em}}\omega = -\frac{1}{T_e T_{em}}\frac{U}{c_1}. \tag{5}$$

Let us henceforth denote respectively by $x_H$ and $\omega_H$ the motor turning angle and its velocity at the beginning of the movement before braking starts; and by $x_K$ and $\omega_K$ the corresponding quantities for the second, final interval during dynamic braking.

Let us first examine the transient process when a step voltage is connected to the system input at instant $t = 0$.

Let us assume that at the moment the signal was applied the system was in the state of rest, i.e., $\omega_H = 0$ and $\frac{d\omega_H}{dt} = 0$ at $t = 0$. The solution of equation (4') under these conditions will take the form

$$\omega_H = \frac{p_2 U e^{p_1 t}}{(p_1 - p_2)c_1} - \frac{p_1 U e^{p_2 t}}{(p_1 - p_2)c_1} + \frac{U}{c_1}, \tag{6}$$

where

$$p_1 = -\frac{1}{2T_e} + \sqrt{\frac{1}{4T_e^2} - \frac{1}{T_e T_{em}}}, \tag{7}$$

$$p_2 = -\frac{1}{2T_e} - \sqrt{\frac{1}{4T_e^2} - \frac{1}{T_e T_{em}}}\,^{*} \tag{8}$$

The motor turning angle at the starting interval will be

$$x_H = \int_0^{t_H} \omega_H \, dt. \tag{9}$$

Where $t_H$ denotes the starting interval duration. By using equation (6), we obtain

$$x_H = \frac{p_2 e^{p_1 t_H} U}{p_1(p_1 - p_2)c_1} - \frac{p_1 e^{p_2 t_H} U}{p_2(p_1 - p_2)c_1} + \frac{U}{c_1}t_H - \frac{p_2 U}{p_1(p_1 - p_2)c_1} + \frac{p_1 U}{p_2(p_1 - p_2)c_1}. \tag{10}$$

We find the turning angle and motor speed in the second interval by integrating (5).

Let us denote the motor shaft angle and motor speed deviations from the required values at instant $t_H$, when the voltage polarity at the armature is reversed, by $x_H$ and $\omega_H$, respectively. We shall then obtain

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* Below we shall only examine systems for which $p_1$ and $p_2$ are real quantities ($T_{em} \geq 4T_e$).

for the starting instant of the second interval when $t = 0$, $\frac{d\omega K}{dt} = 0$ *, $\omega_H = \omega_K$ and $x_K = 0$

$$\omega_{\text{K}} = \left(\omega_{\text{n}} + \frac{U}{c_1}\right)\frac{p_2 e^{p_1 t_{\text{K}}}}{p_2 - p_1} - \frac{p_1}{p_2 - p_1}\left(\omega_{\text{n}} + \frac{U}{c_1}\right)e^{p_2 t_{\text{K}}} - \frac{U}{c_1}, \tag{11}$$

$$x_{\text{K}} = \int\limits_0^{t_{\text{K}}} \omega_{\text{K}} dt. \tag{12}$$

Where $t_K$ denotes the duration of the second interval. By integrating (12), we obtain:

$$x_{\text{K}} = \frac{p_2\left(\omega_{\text{n}} + \frac{U}{c_1}\right) e^{p_1 t_{\text{K}}}}{p_1(p_2 - p_1)} - \frac{p_1\left(\omega_{\text{n}} + \frac{U}{c_1}\right) e^{p_2 t_{\text{K}}}}{p_2(p_2 - p_1)} - \frac{U}{c_1} t_{\text{K}} - \frac{p_2\left(\omega_{\text{n}} + \frac{U}{c_1}\right)}{p_1(p_2 - p_1)} + \frac{p_1\left(\omega_{\text{n}} + \frac{U}{c_1}\right)}{p_2(p_2 - p_1)}. \tag{13}$$

The obtained solutions of the equations will be required below for deriving the system time characteristics.

## 3. Control Unit Block Schematic

Let us now examine the arrangement of an optimum system control unit on whose input is impressed, as distinct from the previously examined case, a signal of frequency $\omega_3$.

In the first place let us find the relationship between the velocity and position balances, at which the control unit must reverse by means of relay $P_1$ the voltage polarity at the armature.

In the second interval when dynamic braking is applied the system will deviate by angle $\bar{x}_K$ equal to

$$\bar{x}_{\text{K}} = x_{\text{n}} + \omega_3 t_{\text{K}} + \omega_{\text{n}} t \tag{14}$$

The second right-hand side term of this expression accounts for the change of the input signal during the second interval $t_K$. The rate of change of the input signal $\omega_3$ over a small interval of time $t_K$ is assumed to be constant.

Equation (14) third right-hand side term accounts for relay P (Fig. 1) operation time $t_{op}$. It is often impossible to ignore this quantity since it is of the same order as the transient state duration.

Considering that when $t = t_K$, $\omega = \omega_3$, we obtain

$$\omega_3 = \left(\omega_{\text{n}} + \frac{U}{c_1}\right)\frac{p_2 e^{p_1 t_{\text{K}}}}{p_2 - p_1} - \frac{p_1}{p_2 - p_1}\left(\omega_{\text{n}} + \frac{U}{c_1}\right)e^{p_2 t_{\text{K}}} - \frac{U}{c_1}. \tag{15}$$

With the help of (13) and (14), we obtain

$$\begin{aligned} x_{\text{n}} + \omega_3 t_{\text{K}} + \omega_{\text{n}} t_{op} = {} & \frac{p_2\left(\omega_{\text{n}} + \frac{U}{c_1}\right) e^{p_1 t_{\text{K}}}}{p_1(p_2 - p_1)} - \frac{p_1\left(\omega_{\text{n}} + \frac{U}{c_1}\right) e^{p_2 t_{\text{K}}}}{p_2(p_2 - p_1)} - \\ & - \frac{U}{c_1} t_{\text{K}} - \frac{p_2\left(\omega_{\text{n}} + \frac{U}{c_1}\right)}{p_1(p_2 - p_1)} + \frac{p_1\left(\omega_{\text{n}} + \frac{U}{c_1}\right)}{p_2(p_2 - p_1)}. \end{aligned} \tag{16}$$

In this case it is impossible to find the required relationship between $x_{\Pi}$ and $\omega_{\Pi}$ by the method suggested in [1] since velocity $\omega_3$ at the end of the interval is not zero. Hence, for obtaining this relationship from (15) and (16) let us utilize a computer circuit shown in Fig. 2.

This circuit controls relays P and $P_1$ (Fig. 1) according to the input signals which are proportional to the required and actual values of the velocity and deviation angle. The circuit consists of four operational amplifiers $Y_1$ - $Y_4$ [11], used for amplifying, adding and measuring the polarity of the input voltages, of two multiplying

* This equality is valid because at the instant the relay contacts are switched the current stops flowing through the armature. It is shown in [10] that with the adopted method of switching the transient state duration is decreased. For the same reason the optimum process under consideration in this system is represented by two integrals only,although the system contains a third order restriction.

links 1ML and 2ML [12] which multiply the input voltages, and of two nonlinear transducers 1NT and 2NT [13] whose output voltages are nonlinear functions of their input voltages.* The values of the voltages in various parts of the circuit are denoted by the same letters as the variables in equations (15) and (16). Thus, $\alpha$ denotes the sum $\omega_{\text{П}} + \dfrac{U}{c_1}$.

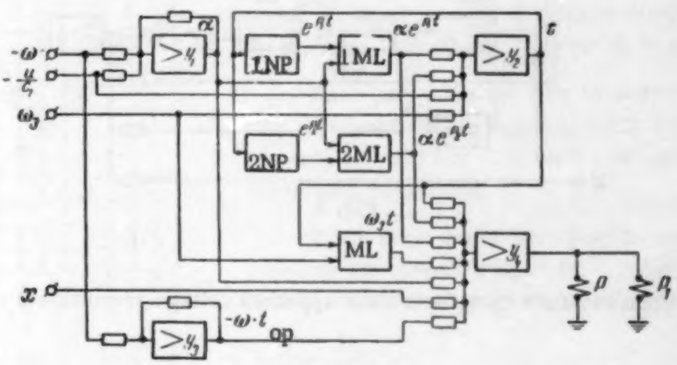


Fig. 2

At the output of amplifier $Y_2$ there appears a voltage proportional to the minimum time required for making $\omega = \omega_з$. The term for correcting the error due to the relay operation time $\omega t_{op}$ appears at the output of amplifier $Y_3$ as the result of multiplying velocity $\omega$ by the amplifier $Y_3$ fixed gain, equal to the relay operating time $t_{op}$. In order to make this circuit control the motor two relays P and $P_1$ are connected to the circuit output.

In certain cases the electromechanical time constant $T_{em}$ is considerably larger than the electrical one $T_e$, and the latter can be neglected.** The computer control unit circuit then becomes considerably simpler.

At $T_e = 0$ instead of (15) and (16), we obtain

$$\omega_e = \left(\omega_{\text{п}} + \frac{U}{c_1}\right) e^{-\frac{t_{\text{к}}}{T_{em}}} - \frac{U}{c_1}; \tag{17}$$

$$x_{\text{п}} + \omega_з t_{op} + \omega_{\text{п}} t_{op} = T_{em}\left(\omega_{\text{п}} + \frac{U}{c_1}\right) e^{-\frac{t_{\text{к}}}{T_{em}}} + T_{em}\left(\omega_{\text{п}} + \frac{U}{c_1}\right) - \frac{U}{c_1} t_{\text{к}}. \tag{18}$$

With the help of (17) and (18) we obtain

$$x_{\text{п}} = T_{em}\left\{(\omega_{\text{п}} - \omega_з) + \left(\omega_з + \frac{U}{c_1}\right)\left[\operatorname{sign}\omega_з \ln\left(|\omega_з| + \frac{U}{c_1}\right) - \right.\right.$$
$$\left.\left. - \operatorname{sign}\omega_{\text{п}} \ln\left(|\omega_{\text{п}}| + \frac{U}{c_1}\right)\right] - \omega_{\text{п}} \frac{t_{op}}{T_{em}}\right\}. \tag{19}$$

Where sign $\omega_з$ and sign $\omega_{\text{П}}$ show that with the change of the signs of $\omega_з$ and $\omega_{\text{П}}$ the signs of $\ln\left(|\omega_з| + \frac{U}{c_1}\right)$; $\ln\left(|\omega_{\text{п}}| + \frac{U}{c_1}\right)$ change accordingly.

The circuit of the optimum system control unit for the case in question is shown in Fig. 3. It includes five operational amplifiers ($Y_1$ - $Y_5$), two nonlinear transducers 1NT and 2NT one multiplying link ML and two

* The method of obtaining circuits for solving transcendental equations is examined in [14].

** The possibility of neglecting $T_e$ can be judged from a graph given below.

relays P and $P_1$ for switching the armature.

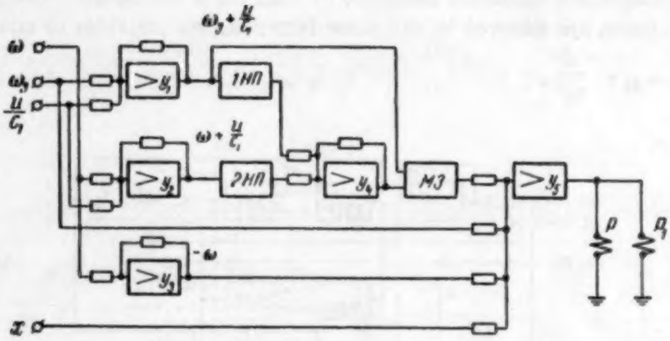


Fig. 3

At the output of the first nonlinear transducer there appears a voltage proportional to sign $\omega_з \ln\left(|\omega_з| + \frac{U}{c_1}\right)$, and at the output of the second one a voltage proportional to sign $\omega_п \ln\left(|\omega_п| + \frac{U}{c_1}\right)$. Such relations can be obtained fairly simply by means of carborundum resistors (tyrite and vilite).*

The first and last terms of the figure brackets of expression (19) are represented in the form of a product $\omega_п (1 - \frac{t_{op}}{T_{em}})$ which is formed in the computer by one of the resistors at the input of $Y_5$.

The operational amplifiers $Y_1 = Y_5$ are shown in Fig. 3 only for the purpose of making it simpler to follow the calculation carried out according to (19).

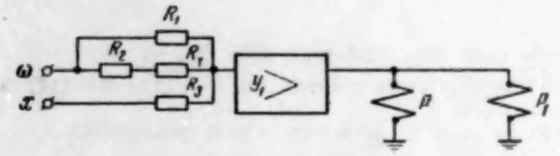

Fig. 4

This circuit can be simplified considerably and need not contain above amplifiers; their functions can be taken over by the nonlinear links 1NT, 2NT, and ML.

Finally, if it is assumed that the system operates on a step-by-step unbalance only, $\omega_з = 0$ and the values of the velocity and angle unbalance will be related by the expression

$$x_п = T_{em} \frac{U}{c_1}\left[\frac{c_1\omega_п}{U} - \operatorname{sign}\omega_п \ln\left(\left|\frac{\omega_п c_1}{U}\right| + 1\right)\right] \tag{20}$$

or taking into consideration the relay operating time

$$x_п = T_{em} \frac{U}{c_1}\left[\frac{c_1\omega_п}{U} - \operatorname{sign}\omega_п \ln\left(\left|\frac{\omega_п c_1}{U}\right| + 1\right) - \frac{c_1\omega_п t_{op}}{UT_{em}}\right]. \tag{21}$$

The control unit circuit for this case is shown in Fig. 4. It includes amplifier $Y_1$, tyrite $R_T$, resistor $R_2$, connected in series with the tyrite and resistors $R_1$ and $R_3$. Resistor $R_2$ is connected in series with the tyrite in order to make its nonlinear characteristic close to the required one. Resistor $R_1$ is included in order to compensate the effect produced by the relays P and $P_1$ operating time $t_{op}$ on the control signal.

These relays, in a manner similar to the preceding circuits, are meant to control the servo motor.

* Carborundum resistors are also used for an exact and simple representation of the relation $y = x\,|x|$, required for optimum systems examined in [1].

## 4. Graph for Choosing a Power Unit

The design of servosystems is usually started by choosing a power unit. For a correct choice of a motor (or any other type of power unit) it is necessary to know the relation between the load, the motor parameters and the response to at least one typical, for instance, step-by-step input signal.

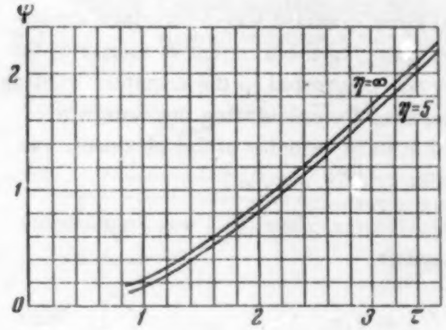


Fig. 5

It was stated above that such relationships could be obtained for certain types of optimum servo systems and represented, for instance, in the form of a family of curves.

Such curves for the type of servo systems under consideration are shown in Fig. 5. They have been plotted on the basis of equations (10) and (13) for the two intervals of the system movement considered.

For a more convenient graphic representation the above relations are given in Fig. 5 in dimensionless units. Along the X-axis was taken the dimensionless response time

$$\tau = \frac{t}{T_{em}}, \tag{22}$$

and along the Y-axis the dimensionless quantity

$$\varphi = \frac{x c_1}{T_{em} U}. \tag{23}$$

Where $t$ is the system response time to a given unbalance and $x$ is the value of the unbalance.

As a parameter for the family of curves serves the quantity

$$\eta = \frac{T_{em}}{T_e}. \tag{24}$$

The curves for $\eta = 5$ and $\eta = \infty$ are close to each other. Hence it is possible to neglect the electrical time constant in this case (roots $p_1$ and $p_2$ are real). Moreover, as it was pointed out before, the control unit circuit is thus considerably simplified (Figs. 3 and 4).

It is possible by means of these curves, on the basis of the given platform moment of inertia and the permissible response time, to choose a required type of motor or to arrive at the conclusion that with the given servo system it is impossible to meet the given technical requirements.

If the type of motor is given, it is possible to determine the response time for the required platform moment of inertia.

By means of the same curves it is possible to determine how close to an optimum is any servosystem. Such comparisons must, of course, be made with similar loading. As the result of these comparisons it is possible to decide whether it is expedient to change the control unit of any given servosystem with a view to improving its speed of operation.

## 5. Experimental Investigation

A most simple optimum servo system, whose circuit is shown in Fig. 4, was investigated experimentally.

The investigation was carried out with a view of checking its efficiency and for the purpose of evaluating the error due to the idealization of the system in its mathematical treatment.*

The model of an optimum servosystem included a dc motor type SL-267, which turned, by means of a reduction gear of ratio $i_p = 120$, a platform with a moment of inertia $J = 0.05$ kg $sec^2$.

* V. P. Grekova took part in the experimental investigation of the model of the system.

A voltage proportional to the velocity of the platform was obtained from tachometer generator whose shaft was connected by means of reduction gear to the motor shaft.

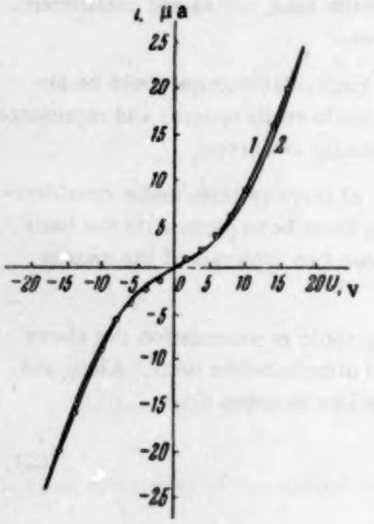


Fig. 6

A voltage proportional to the turning angle of the platform was obtained from the slide of a wire wound potentiometer fixed to the platform shaft.

The electromechanical time constant was determined experimentally at $T_{em} = 0.1$ sec by changing the moment of inertia of the platform. A voltage of 110 v was connected to the armature winding. The nonlinear ratio $U/c_1$, required for calculating the system resistance, and equal to the stable state velocity of the platform, was also determined experimentally at 3.76 $sec^{-1}$.

The voltage at the potentiometer slide ($U_{\Pi}$) was proportional to the turning angle of the platform

$$U_{п} = k_1 x, \tag{25}$$

where $k_2$ = 2 v/degree.

The voltage $U_{TG}$ at the tachometer generator was proportional to the velocity of the platform:

$$U_{TG} = k_2 \omega_1, \tag{26}$$

where $k_2$ = 0.083 v sec/degree.

The switching of the armature was made by the contacts of relay $P_1$ whose winding was connected to the output of the operational amplifier $Y_1$ (Fig. 4). The relay operating time was equal to $t_{op} = 20\ \mu$ sec.

Figure 6 shows the required relation between the velocity and position unbalance at the instant of the armature voltage switching (curve 1). Voltage U proportional to the velocity was plotted along the X-axis and current i proportional to the right-hand side of formula (20) was plotted along the Y-axis. The same graph shows the volt-ampere characteristic of a tyrite (curve 2). The tyrite was in the shape of a cylinder 10 mm high and 50 mm in diameter.

In order to obtain a better agreement between the natural volt-ampere characteristic of a tyrite and the required curve, resistance $R_2$ (Fig. 4) was connected in series with the tyrite. The points of the volt-ampere characteristic of such a combination are shown in Fig. 6. Practically all of them coincide with the required curve. The value of $R_3$ can be chosen on the basis of the following considerations.

Let the switching occur at the maximum velocity $\omega_{\Pi} = \omega_{max} = \frac{U}{c_1}$, which corresponds according to (20) to the position unbalance of

$$x_{\Pi} = T_{em}\,\omega_{max}\,[1 - \ln 2] = 0.31\ T_{em}\omega_{max}. \tag{27}$$

With selected values of $T_{em}$ and $\omega_{max}$, $x_{\Pi} = 0.116$ radians. According to (25) $U_{\Pi} = 13.3$ v and according to (26) $U_{TG} = 17.9$ v.

At the instant of contact switching the voltage at the amplifier input (at point a, Fig. 4) is equal to zero. Hence,

$$\frac{U_{TG}}{R_T + R_2} = \frac{U_{п}}{R_3}. \tag{28}$$

According to Fig. 6 $\frac{U_{TG}}{R_T + R_2} = 24.5 \cdot 10^{-6}$ amp at $\omega_{max}$. Thus, $R_3 = 544$ kohms. It is possible to find the value of $R_1$ which compensates for the relay operating time from (20), (25), and (26):

$$R_1 = \frac{k_3 R_3}{t_{cp} k_1}. \quad (29)$$

With the chosen circuit parameters $R_1 = 1.1$ megohm.

Figure 7 shows oscillograms of the response time of the optimum servosystem under consideration, working with step unbalances of 20, 30, and 40° (oscillograms a, b, and c, respectively).

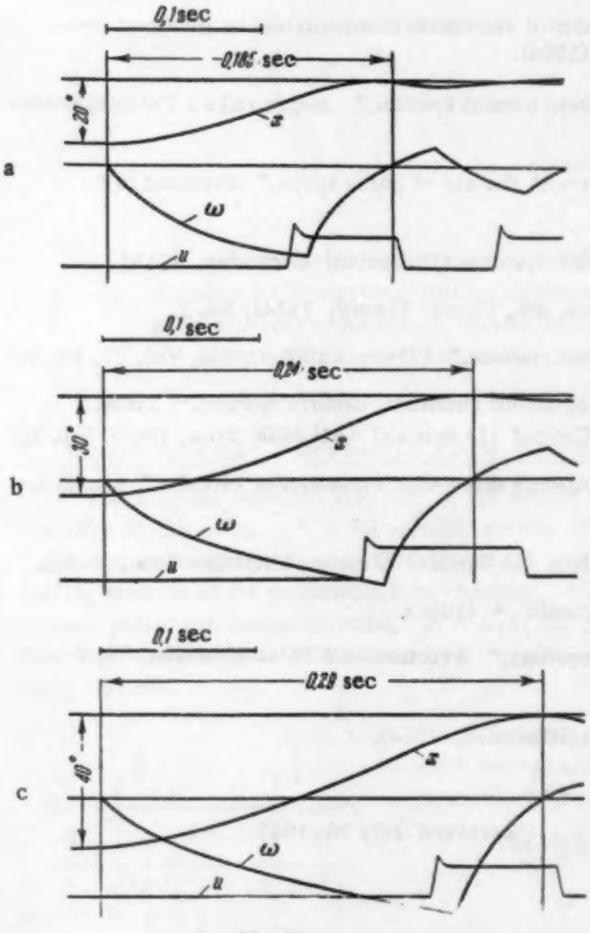


Fig. 7

The oscillograms show the values of the position (x) and velocity (ω) unbalances and the voltage at the amplifier Y (U) output.

At the end of the transition process the position and velocity unbalances became equal to zero. The switching of the relay contacts occurred later than the jump of the amplifier Y output voltage by the amount of $t_{op}$.

When tested the system did not include the linear amplifier shown in the circuit of Fig. 1. Relay $P_1$ (Fig. 1), which disconnects the motor armature at small unbalances, was also missing. That is why at the end of the transition process oscillations were observed.

Let us determine from Fig. 5 the response time of the system and compare it with that obtained from the oscillograms.

According to (23) $\varphi = \frac{xc_1}{UT_{em}} = 0.93$ at $x = 20°$. This value of $\varphi$ corresponds to $\tau = 2.06$ and $t^* = \tau T_{em} = 0.205$ sec.

The relative error in determining the response time by the graph of Fig. 5 as compared with the experiment is equal to $\delta = \frac{t^* - t}{t^*} \approx 10\%$.

Above response time was obtained from the oscillogram of Fig. 7, a. For oscillograms in Fig. 7, b and c similar errors were found to be equal to 8.9% and 8%, respectively.

Thus, for the three cases examined the actual response time did not differ from the calculated one by more than 10%.

If the time constant $T_{em}$ should change in the course of the operation of the system (for instance with time or with the deviation angle), the optimum law of control can be maintained by an automatic variation of resistances $R_1$ and $R_3$ in the control unit.

## SUMMARY

It is expedient to plot curves showing the relation between the speed of operation of an optimum AC system and its power unit parameters for the most common types of power units and loading.

It is possible by means of these curves to find the response time of an optimum ACS which includes any type of power unit, thus, determining the usefulness of the latter.

For the type of optimum systems investigated the response time obtained experimentally differs by not more than 10% from the one found from the curves.

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Received July 30, 1957

* See English translation.

# CALCULATION OF TIME CHARACTERISTICS OF PNEUMATIC FLOW CHAMBERS


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**Nonlinear differential equations of pressure variations in a pneumatic flow chamber are integrated with the assumption that any of the input quantities are subject to step variations.* Examples of time characteristics calculations are given and these results are compared with characteristics obtained experimentally.**


## 1. Equations for Pneumatic Flow Chamber Time Characteristics

The adopted notations are: $Q^*$ is the weight of air in the interthrottle chamber, V is the volume of the interthrottle chamber, $\gamma^*$ is the specific gravity of air in the interthrottle chamber, R is the gas constant, T is the absolute temperature of air, $G^*_i$ is the weight rate of flow of air through the i-th throttle, $P_0$ is the feeding pressure of the pneumatic flow chamber, $P^*_1$ is air pressure in the interthrottle chamber, $P_2$ is the air pressure passed the second throttle, $f_i = \mu_i F_i$ the effective port area of the i-th throttle, $\mu_i$ is the rate of flow coefficient of the i-th throttle, $F_i$ is the port area of the i-th throttle, i is the number of the throttle, and t is time.

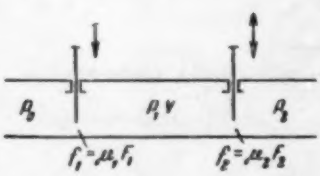


Fig. 1

Let us now deduce a differential equation for pressure $P_1^*$ variations with respect to time in the pneumatic flow chamber (Fig. 1). Let us assume that $T_0 = T_1 = T_2 = T$.

The weight of air in the interthrottle chamber will be

$$Q^* = V\gamma^*. \tag{1}$$

Differentiating equation (1) with respect to time and using the gas equation we obtain

$$\frac{dQ^*}{dt} = \frac{V}{RT}\frac{dP_1^*}{dt}. \tag{2}$$

In equation (2) quantity $dQ^*/dt$ represents the weight rate of flow of air into the interthrottle chamber (or out of it) in a dynamic state, i.e., $dQ^*/dt = G_1^* - G_2^*$.

Taking this into consideration and rewriting equation (2), we obtain

$$\frac{V}{RT}\frac{dP_1^*}{dt} = G_1^* - G_2^*. \tag{3}$$

The weight rates of flow $G_1^*$ and $G_2^*$ can be calculated from formulas [2]:

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* Differential equations of pressure variations in pneumatic device flow chambers with linear approximation are considered in [1].

a) for a pre-critical state of flow

$$G_i^{\bullet} = f_i \sqrt{\frac{2g}{RT} P_i^{\bullet} (P_{i-1} - P_i^{\bullet})}, \quad \frac{P_i}{P_{i-1}} \geqslant 0{,}5; \tag{4a}$$

b) for an above-critical state of flow

$$G_i^{\bullet} = f_i P_{i-1} \sqrt{\frac{g}{2RT}}, \quad \frac{P_i}{P_{i-1}} \leqslant 0{,}5. \tag{4b}$$

It should be noted that there can be four possible combinations of flow through the first and second throttles: "p – p", "a – p", "p – a" and "a – a" (pre-critical flow through the first and second throttles, above-critical flow through the first and pre-critical through the second, etc.).

By substituting in (3) the corresponding expressions in (4a) and (4b) it is possible to obtain four differential equations related to the four combinations of flow. The expressions thus obtained, subject to the condition that any of the input quantities ($P_0$, $f_1$, $f_2$, and $P_2$) changes in a step-by-step manner, become equations of the first order with separable variables and can be integrated. In integrating let us assume that the rate of flow coefficients $\mu_i$ are constant during the transient state. As the result of integration, we obtain:

for combination "p – p"

$$t = \frac{A}{2k}\left\{\operatorname{arc\,tg}\frac{2r_1 - 1}{2\sqrt{r_1(1-r_1)}} - \frac{\beta_1\beta_2}{2\varphi}\left[\operatorname{arc\,tg}\frac{\beta_1^2(2r_1 - 1) + r_1}{2\beta_1\beta_2\sqrt{r_1(1-r_1)}} - \operatorname{arc\,tg}\frac{2r + \beta_1^2 - r_1}{2\sqrt{(r+\beta_1^2)(r_1 - r)}}\right] + \right.$$

$$\left. + \frac{\beta_3\beta_4}{2\varphi}\ln\left|\frac{\beta_3\sqrt{1-r_1} + \sqrt{r_1}\beta_4}{\beta_3\sqrt{1-r_1} - \sqrt{r_1}\beta_4} \cdot \frac{\sqrt{\beta_3^2 - r} + \sqrt{r_1 - r}}{\sqrt{\beta_3^2 - r} - \sqrt{r_1 - r}}\right|\right\} + c, \tag{5*}$$

where $\underline{c}$ is the integration constant which is found from the initial conditions.

$$\varphi = \sqrt{\left(\frac{1}{2} - \frac{r}{2k^2}\right)^2 + \frac{r^2}{k^2}}, \quad A = \frac{V}{f_2}\sqrt{\frac{2}{gRT}},$$

$$k = \frac{f_1}{f_2}, \quad r = \frac{P_2}{P_0}, \quad r_1 = \frac{P_1}{P_0},$$

$$\beta_1 = \sqrt{-\frac{1}{2} + \frac{r}{2k^2} + \varphi}, \quad \beta_2 = \sqrt{\frac{1}{2} + \frac{r}{2k^2} + \varphi},$$

$$\beta_3 = \sqrt{\frac{1}{2} - \frac{r}{2k^2} + \varphi}; \quad \beta_4 = \sqrt{\frac{1}{2} + \frac{r}{2k^2} - \varphi};$$

for combination "a – p"

$$t = A\left[-\sqrt{\frac{1}{r_2} - 1} - \frac{k}{2r}\ln\left|\sqrt{\frac{1}{r_2} - 1} - \frac{k}{2r}\right|\right] + c, \tag{6}$$

where

$$r_2 = P_2/P_1^{\bullet};$$

for combination "p – a"

$$t = A\frac{2}{4k^2 + 1}\left[2k\operatorname{arc\,tg}\sqrt{\frac{r_1}{1 - r_1}} - \ln\left|\sqrt{r_1} - 2k\sqrt{1 - r_1}\right|\right] + c; \tag{7}$$

and for combination "a – a"

$$t = A\ln|k - r_1| + c. \tag{8}$$

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* Arc tg = $\tan^{-1}$ and ln = $\log_e$ – Publisher's note.

In the case when $f_1$ or $f_2$ become zero, expressions (5) - (8) degenerate respectively into equations representing the emptying and filling of the dead-end chamber.

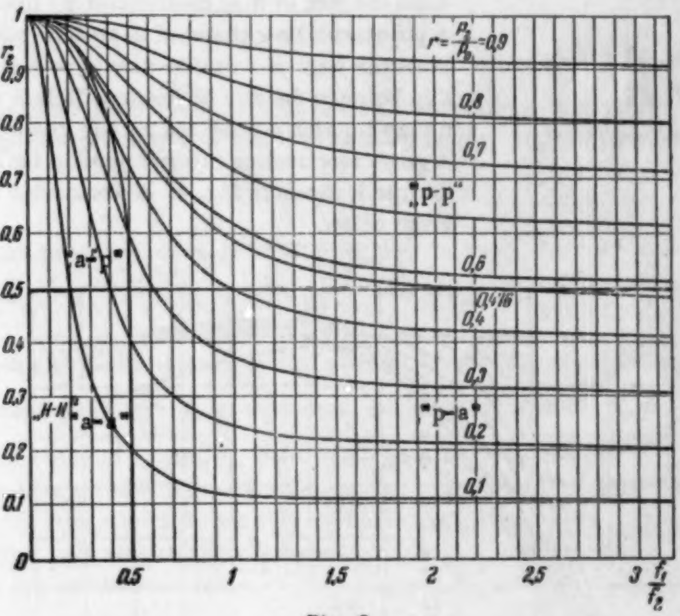


Fig. 2

Case when $f_1 = 0$:

a) pre-critical flow

$$t = -A\sqrt{\frac{1}{r_2} - 1} + c; \tag{9}$$

b) above-critical flow

$$t = -A \ln \frac{1}{r_2} + c. \tag{10}$$

Case when $f_2 = 0$:

a) pre-critical flow

$$t = A_1 \operatorname{arc\,tg} \sqrt{\frac{r_1}{1 - r_1}} + c \tag{11}$$

where

$$A_1 = \frac{V}{f_1}\sqrt{\frac{2}{gRT}};$$

b) above-critical flow

$$t = A_1 r_1 + c. \tag{12}$$

It is interesting to note that the transient processes in the interthrottle chambers do not depend on the absolute values of $P_0$, $P_1^*$, and $P_2$, but on their ratios $r_1$, $r_2$, and $\underline{r}$. For calculating the transient process in the interthrottle chamber it is necessary to know beforehand through what flow combinations the process will pass (and it can, as we shall see below, extend to three flow combinations through the first and second throttles), and to what state of flow the initial static state corresponds and what are its parameters. It is impossible to solve these problems by means of static equations for a pneumatic flow chamber, since if $P_1^*$ is unknown, the combination of flows through the first and second throttles is also unknown, and it is impossible to decide which of the static equations should be used (corresponding to "p − p", "p − a", "a − p", or "a − a" combinations).

This problem is solved by means of curves in Fig. 2 plotted from static equations for a pneumatic flow chamber.*

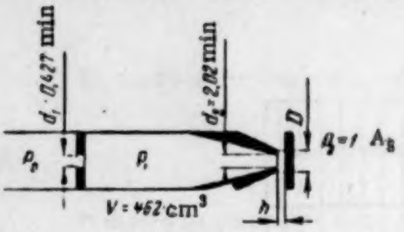


Fig. 3

In addition when calculating transient processes in the inter-throttle chamber of a pneumatic device (Fig. 1) it is necessary to know the rate of flow coefficients $\mu_1$ through the throttle devices. A pneumatic flow chamber in an auto-pneumatic devices usually takes the form of a nozzle-vane type relay (Fig. 3). The relationship between the rate of flow coefficient and the throttle parameters for such a relay are usually given in a form of a graph. A graph representing this relationship for a nozzle-vane type throttle is shown in Fig. 4. A description of two such graphs is given below.

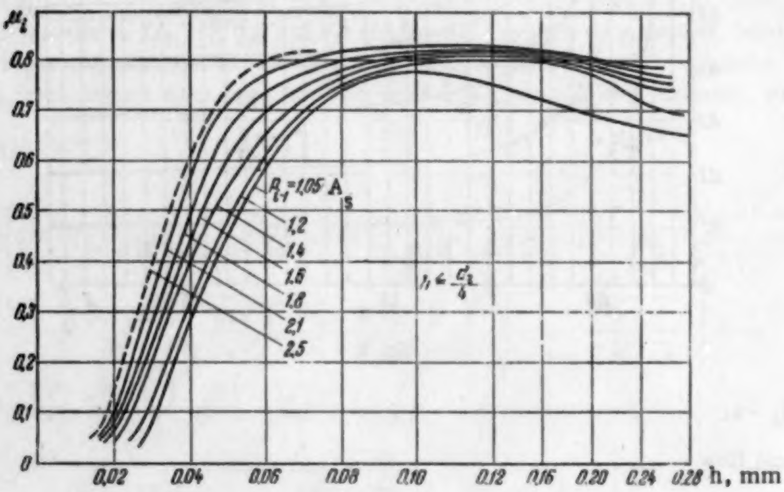


Fig. 4

## 2. Auxiliary Graphs

Figure 2 shows a graph by means of which it is possible to determine the initial static state parameters, the combination of flows at the initial static state, the combination of flows through the first and second throttles during the transient process and the combination of flows at the new stable state value of the variable parameter. The graph is split-up into four zones, corresponding to the four possible combinations of flow through the first and second throttles.

The equations shown in Fig. 2 are obtained from the condition of equality of static weight flows [expressions (4a) and (4b)] through the first and second throttles for various flow combinations. These equations have the form:

for the region of combinations "p – p"

$$\left(\frac{f_1}{f_2}\right)^2 = r_2 r \,\frac{1 - r_2}{r_2 - r}; \tag{13}$$

for the region of combinations "a – p"

$$\left(\frac{f_1}{f_2}\right)^2 = 4r^2\left(\frac{1 - r_2}{r_2}\right);$$

for the region of combinations "p – a"

* A similar graph for Saint-Venant-Wentzel formulas is given in [3].

$$\left(\frac{f_1}{f_2}\right)^2 = \frac{r}{4(r_2 - r)};$$

for the region of combinations "a – a"

$$\frac{f_1}{f_2} = \frac{r}{r_2}.$$

An equation for the boundary between regions of flow combinations "p – p" and "a – p" can be found if in equation (13) $\underline{r}$ is substituted by $r_1 r_2$ and it is assumed that $r_1 = 0.5$. After this transformation we obtain $\left(\frac{f_1}{f_2}\right)^2 = r_2(1 - r_2)$.

The equation for the boundary between the flow regions "a – p", "a – a" and "p – p", "p – a" is $r_2 = 0.5$ and between regions "a – a" and "p – a" it is $\frac{f_1}{f_2} = 0.5$.

It will be seen from Fig. 2 that in order to find pressure $P_1$ from the graph the ratio of effective areas $\frac{f_1}{f_2}$ and the ratio of pressures $r = \frac{P_2}{P_0}$ must be known. For a given value of $\underline{r}$ and $\frac{f_1}{f_2}$, it is possible to find from the graph the value of $r_1 = P_2/P_1$, from which with the knowledge of $P_2$ it is possible to find $P_1$. This also determines the flow combination which corresponds to the initial static state. It is possible to determine from the graph the flow combinations covered by the transitional state. Thus, with $r = P_2/P_0 = \text{const}$ and a step-by-step changing port area, one should follow the given $\underline{r}$ curve from the point of the initial static state to the point of the new stable state value.

Thus, the regions covered by the transient characteristic can be found and, hence, the equations which should be used for its calculation. The points of intersection of curve $r = \text{const}$ with the region boundary lines will provide the boundary values for quantities $r_2$ and $r_1$.

In a similar manner it is possible to find the air flow combinations through the throttling elements with a step-by-step variation of $P_0$ and $P_2$ at $f_1/f_2 = \text{const}$, with the one difference that in this case the various flow-state regions should be traversed vertically.

Figure 4 shows the relationship between the rate of flow coefficient $\mu_i$, the distance $\underline{h}$ from the nozzle to the vane and the pressure $P_{i-1}$ in front of the nozzle of the nozzle-vane type throttle. The rate of flow coefficients $\mu_i$ have been determined experimentally [4] * and include a range of nozzle diameters from $d_2 = 0.3$ mm to $d_2 = 3$ mm at $T = 239°$ K, $P_2 = 1$ $A_s$ * and $D/d_2 = 1.6$ to 6.5.

## 3. Examples

In order to illustrate the technique of calculating the time characteristics of pneumatic relays of the type under consideration by means of the obtained formulas and graphs we shall give four examples. An actual pneumatic relay of the nozzle-vane type with the dimensions shown in Fig. 3 was taken for an example. Time characteristics, with a step-by-step variation of the second throttle device port area and the feeding pressure $P_0$, were obtained experimentally for this relay, thanks to which it was possible to compare the calculated and experimental time characteristics.

Example 1. The given data was: $P_0 = 2.5$ $A_s$, $P_2 = 1$ $A_s$**, $T = 293°$ K (Fig. 3). The distance between the nozzle and the vane changes in a step of $h_0 = 0.037$ mm to $h_{stable} = 0.083$ mm. It is required to derive a time characteristic of the pneumatic relay $P_1 = f(t)$.

Before attempting to derive the transitional process it is necessary to determine the pressure $P_{10}$ in the interthrottle chamber at the initial static state and the pressure $P_{1\,stable}$ at the new stable state condition. In connection with the fact that the rate of flow coefficient $\mu_2$ for the second throttle device is not constant and depends with a constant opening $\underline{h}$ from the pressure in front of and behind the nozzle, the values of the above pressures should be obtained by the method of consecutive approximations.

* The dotted line has been obtained by extrapolation.

** Here and hereinafter: $A_s$ = absolute pressure.

Let us find the value of $P_{10}$. Taking into consideration that at the initial static state $h_0 = 0.037$ mm and the pressure in the interthrottle chamber lies between the limits of $2.5\ A_s \geq P_{10} \geq 1\ A_s$, let us take as the first approximation $\mu_2^{I} = 0.4$ (Fig. 4). The constant throttle consists of a hole in thin wall, hence it can be assumed that $\mu_1 = \text{const} = 0.8$ [4].

Let us find the ratio

$$\left(\frac{f_1}{f_2}\right)^{I} = \mu_1 \frac{\pi d_1^2}{4} \Big/ \mu_2 \pi d_2 h_0 = 1{,}22$$

and by means of the graph in Fig. 2, considering that $r = 0.4$, find $r_2 = 0.47$ and hence $P_{10} = 2.12\ A_s$.

From the graph Fig. 4 we find a more accurate value for $\mu_2^{II} = 0.48$. Then $(f_1/f_2)^{II} \approx 1$ and hence $P_{10} = 2\ A_s$.

Let us find the stable value of $P_{1\,stable}$. As a first approximation for $h_{stab} = 0.083$ mm we find from graph Fig. 4 that $\mu_2^{I} = 0.8$ and determine $(f_1/f_2)^{I} = 0.218$. By means of graph Fig. 2 we find $r_2 = 0.9$ and $P_{1\,stab}^{I} = 1.08\ A_s$. Using the graph Fig. 3 again we obtain a more accurate value for $\mu_2^{II} = 0.755$ and calculate $(f_1/f_2)^{II} = 0.29$.

From graph Fig. 2 we determine $r_2 = 0.877$ and hence $P_{1\,stab} = 1.14\ A_s$.

The value of $k$ is equal to 0.29, i.e., to the ratio $f_1/f_2$ in the newly established stable state. Let us also calculate $f_2$ for the new state, remembering that $\mu_2 = 0.755$; $f_2 = \mu_2 \pi d_2 h_0 = 0.004\ cm^2$.

Now we have all the data required for calculations. It only remains to determine the flow combinations of the first and second throttles covered by the transient state. For this purpose let us use the graph of Fig. 2. Marking off on curve $r = 0.4$ the point of the initial static state ($r_2 = 0.5$, $r_1 = 0.8$) and the point of the new stable state ($r_2 = 0.877$) we note that the time characteristic will pass through flow combination "p – p" at first and then through "a – p", and the boundary value will be $r_2 = 0.8$ which corresponds to $r_1 = 0.5$.

In the first section ("p – p") calculations are made according to formula (5). The integration constant is determined from the condition $r_1 = 0.8$ and $t = 0$. In the second section ("a – p") calculations should be made by formula (6) and the integration constant determined from condition $r_2 = 0.8$ and $t = t_1$. Here $t_1$ is the time of transition from the flow combination region "p – p" to that of "a – p". The calculation data is given in Table 1.

TABLE 1

| "p-p" | | | "a-p" | | |
|---|---|---|---|---|---|
| $t$, sec | $r_1$ | $P_1 = 2.5\ r_1$, $A_s$ | $t$, sec | $r_2$ | $P_1 = \frac{1}{r_2} \cdot A_s$ |
| 0 | 0.800 | 2.000 | $t_1 = 6.72$ | 0.800 | 1.250 |
| 1.13 | 0.700 | 1.750 | 7.25 | 0.815 | 1.227 |
| 2.76 | 0.600 | 1.500 | 7.84 | 0.830 | 1.205 |
| 3.91 | 0.550 | 1.375 | 8.94 | 0.850 | 1.176 |
| $t_1 = 6.72$ | 0.500 | 1.250 | 12.25 | 0.877 | 1.140 |

$V = 462\ cm^2$ $G = 980\ cm/sec^2$ $R = 2930$ cm/degree
$T = 293°$ K, $k = 0.29$, $r = 0.4$, $f_2 = 0.004\ cm^2$

Figure 5, a gives the experimental curve of the transitional process. It also shows the calculated points (from Table 1). It will be seen from Fig. 5, a that the calculated points agree well with the experimental curve.

Example 2. As distinct from the first example, the distance between the nozzle and the vane this time changes in the opposite direction, i.e., jumping from $h_0 = 0.083$ to $h_{stab} = 0.037$ mm. It is required to determine the time characteristic.

In this example pressure $P_{10}$ in the interthrottle chamber at the initial static state will correspond to pressure $P_{1\,stab}$, determined in the previous example, and the pressure at the new stable state will correspond to that of $P_{10}$, also determined in the previous example. The rate of flow coefficient $\mu_2$ of the second throttle should be taken at its maximum,* equal to $\mu_2 = 0.48$ (Fig. 3). Then, we shall obtain $k = 1$, and $f_2 = 0.00113$ cm$^2$.

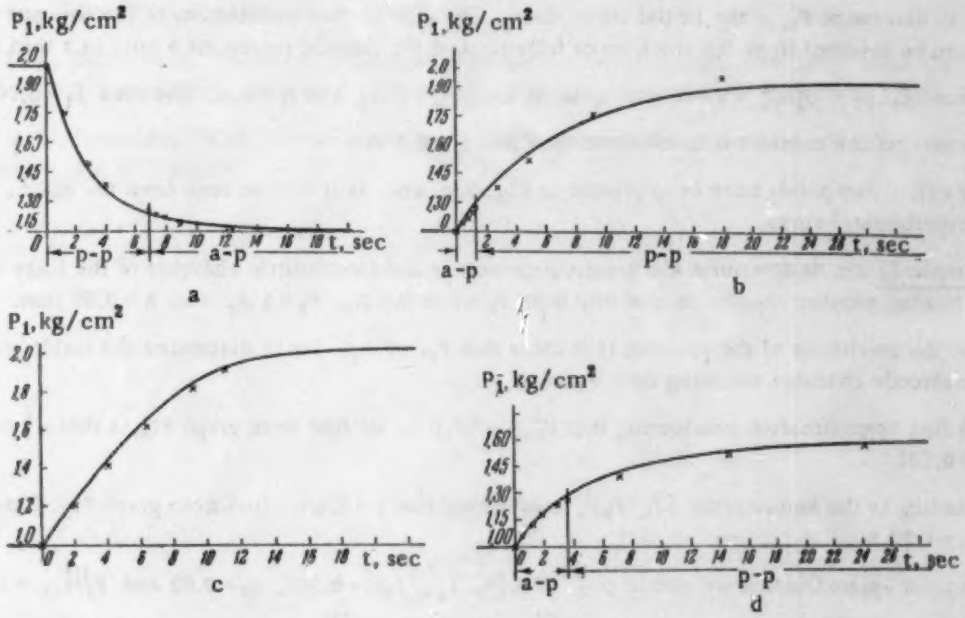


Fig. 5

Integration constants are determined from the condition that for "a – p" $r_2 = 0.877$ at $t = 0$ and for "p – p" $r_1 = 0.5$ at $t = t_1$.

All the calculation data is given in Table 2.

TABLE 2

| "a-p" | | | "p-p" | | |
|---|---|---|---|---|---|
| $t$, sec | $r_2$ | $P_1 = \frac{1}{r_2}$, atm | $t$, sec | $r_1$ | $P_1 = 2.5\, r_1$ atm |
| 0 | 0.877 | 1.140 | $t_1 =$ 1.36 | 0.500 | 1.250 |
| 0.40 | 0.850 | 1.176 | 2.87 | 0.550 | 1.375 |
| 0.78 | 0.830 | 1.205 | 4.85 | 0.600 | 1.500 |
| 1.04 | 0.815 | 1.227 | 9.17 | 0.700 | 1.750 |
| $t_1 =$ 1.36 | 0.800 | 1.250 | 18.00 | 0.800 | 2.000 |

The calculated points are plotted in Fig. 5,b together with the corresponding experimental curve. The maximum deviation of a calculated point from the experimental curve is equal to 7%.

* In view of the fact that $\mu_2$ is a function of the pressure in front of the nozzle, it would have been more correct in this case to have divided the whole range of $P_1$ variations into sections and taken the mean value of the rate of flow coefficient for each section. If, for the sake of simplicity, however, only one value of $\mu_2$ is chosen, it should be the maximum (in this instance corresponding to the end of the transition process), since only in this case will the calculated value approach the experimental one. Physically this will mean that instead of taking the effective port area $f_2$ as increasing gradually during the transition period it is taken at its maximum value. It can therefore be expected that the calculated points at the beginning of the transitional process will lie below the experimental curve.

Example 3. The distance between the nozzle and the vane is changed in one step from $h_0 = \infty$ to $h_{stab} = 0$. The feeding pressure is $P_0 = 2$ $A_s$, $P_2 = 1$ $A_s$, T = 293° K. It is required to determine the time characteristic.

It will be seen from above condition that in this instance it is the question of filling the dead-end chamber. Since r = 0.5 (Fig. 3) the flow into the chamber is pre-critical. The calculation should be made according to formula (11).

Let us determine $P_{10}$ at the initial static state. The rate of flow coefficients of the first and second throttles can be assumed to be 0.8 since when fully opened the throttle represents a hole in a thin wall.

Hence $f_1/f_2 = d_1^2/d_2^2 = 0.045$ and $r_2 \approx 1$, i.e., $P_{10} = 1$ $A_s$, and $r_1 = 0.5$. The area $f_1 = 0.00115$ cm$^2$.

The integration constant is found from condition $r_1 = 0.5$ at t = 0.

The calculated points have been plotted in Fig. 5, c and, as it will be seen from the graph, agree well with the experimental curve.

Example 4. Let us determine the transient process in the interthrottle chamber of the relay (Fig. 3), when the feeding pressure changes in one step from $P_0 = 1$ to 2.5 $A_s$, $P_2 = 1$ $A_s$ and h = 0.05 mm.

From the conditions of the problem it is clear that $P_{10} = 1$ $A_s$. Let us determine the stable state pressure in the interthrottle chamber assuming that $\mu_1 = 0.8$.

As a first approximation considering that $P^{I}_{1stab} = 1.2$ $A_s$ we find from graph Fig. 4 that $\mu_2^{I} = 0.4$ and $(f_1/f_2)^{I} = 0.737$.

According to the known ratio $(f_1/f_2)^{I}$, considering that r = 0.4, we find from graph Fig. 2 that $r_2 = 0.58$ and $P^{II}_{1stab} = 1.72$ $A_s$.

As a third approximation we obtain $\mu_2^{III} = 0.64$, $(f_2/f_2) = 0.565$, $r_2 = 0.68$ and $P^{III}_{1stab} = 1.47$ $A_s$.

As the fourth approximation we obtain $\mu_2^{IV} = 0.58$, $(f_1/f_2)^{IV} = 0.625$, $r_2 = 0.64$ and $P_{1stab} = 1.56$ $A_s$. The value of k is the one determined in the last approximation by ratio $f_1/f_2 = 0.625$.

Considering that $\mu_2 = 0.58$ we find $f_2 = \mu_2 \pi d_2 h_{stab} = 0.00184$ cm$^2$.

In order to establish the flow combination through the first and second throttles at the transient process it is necessary to take into account that at the point of the initial static state $r_2 = 1$, r = 1 and $f_1/f_2 = 0$, and in the new stable state $r_2 = 0.64$ and r = 0.4.

By following the curve r = 0.4, from the first point to the second it is seen that the initial stage of the transient process will occur in "a – p", the subsequent stage in "p – p" and the boundary value of $r_2 = 0.8$. The calculation of the transitional process should therefore be carried out according to formulas (6) and (5).

The integration constant for the case "a – p" is obtained from condition $r_2 = 1$ at t = 0, and for the case "p – p" from condition $r_1 = 0.5$ at $t = t_1$.

All the calculation results are given in Table 3.

TABLE 3

| "a-p" | | | "p-p" | | |
|---|---|---|---|---|---|
| $t$, sec | $r_2$ | $P_1 = \frac{1}{r_2}$, $A_s$ | $t$, sec | $r_1$ | $P_1 = 2.5\,r_1$, $A_s$ |
| 0 | 1.000 | 1.000 | $t_1 =$ 3.65 | 0.500 | 1.250 |
| 0.52 | 0.950 | 1.052 | 7.22 | 0.550 | 1.375 |
| 1.24 | 0.900 | 1.110 | 14.81 | 0.600 | 1.500 |
| 1.98 | 0.850 | 1.180 | 24.23 | 0.620 | 1.550 |
| 3.02 | 0.820 | 1.220 | 33.03 | 0.625 | 1.562 |
| $t_1 =$ 3.65 | 0.800 | 1.250 | | | |

Points found by calculation are plotted in Fig. 5, d. The same graph for comparison carries the experimental curve.

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Received January 13, 1958

* See English translation.

# EMERGENCY CONNECTION OF LAMPS

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A method of emergency connection of lamps (electric supply, lighting, etc.) by placing them in parallel is examined and the required values of the voltage and ballast resistance are determined for the condition that when one of the lamps fails the resulting optical and emission characteristics remain unchanged.

Many automatic devices use incandescent lamps and electron tubes. The electron tubes are used as amplifiers, oscillators, detectors, relays, etc.; incandescent lamps are used as sources of light for photoelectric amplifiers, photorelays, photoindicators, etc.

Since the lamp filament under the action of the heating current wears and burns out in time, emergency connection of lamps become necessary. The simplest method of emergency connection is the parallelling of two lamps. A simple parallel connection however, is not suitable, since the failure of one filament changes the total anode current in the case of tubes. Similarly in case of incandescent lamps the resulting luminous flux is changed. Above circumstances lead to a search for other methods of emergency connection.

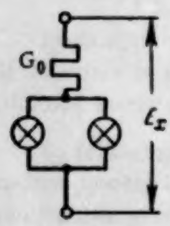


Fig. 1

A constant anode current or luminous flux when one of the parallel filaments fails in the case of electron tubes and incandescent lamps, respectively, can be assured by means of the circuit connection shown in Fig. 1.

Let us examine the connection of two lighting or signalling lamps with the view of increasing the reliability of their work. Let there be a fixed condition that the lighting of the surface must remain constant whether one of two lamps are alight. Let us attempt to find the required condition of operation for the lamps.

Figure 2 shows the relation between the luminous flux and voltage across a lamp $\Phi = f\,(U)$ [more accurately $\Phi/\Phi_{100} = f\,(U/U_{100})$] in the form of curve $\Phi$. The same graph shows the relation between the current I through the filament and the voltage across the lamp [more accurately $I/I_{100} = f\,(U/U_{100})$] in the form of curve I. The graph also includes curves for the luminous flux and current for two lamps (curves $2\Phi$ and 2I). Here $U_{100}$ and $\Phi_{100}$ denote the nominal values of voltage and luminous flux for a lamp.

Let us assume that a luminous flux $\Phi_x$ is required from the lamps. Let us find on the curve $2\Phi$ point a, corresponding to the required flux $\Phi_x$ (or $\Phi_x/\Phi_{100}$). It corresponds to voltage $U_a$ (or $U_a/U_{100}$). The same voltage $U_a$ corresponds to a current through the lamps determined by point a' on curve 2I.

If one of the lamps burns out, the luminous flux according to the operation condition, must remain constant. For the value $\Phi_x$ we obtain on curve $\Phi$ a point b. It corresponds to voltage $U_b$ (or $U_b/U_{100}$) at which the current through the lamp is determined by point b' on curve I.

In order to obtain these relationships the lamps should be connected as shown in Fig. 1. In order to find the value of resistor $r_0$ and that of the supply voltage $E_x$ let us draw a straight line through point a and b. This

line will cross the X-axis at point c which determines the required voltage $E_x$ (or $E_x/U_{100}$). Resistance $r_0$ is determined from condition $\frac{E_x - U_b}{I_b} = r_0$ (where $U_b$ and $I_b$ correspond to point b' on curve I).

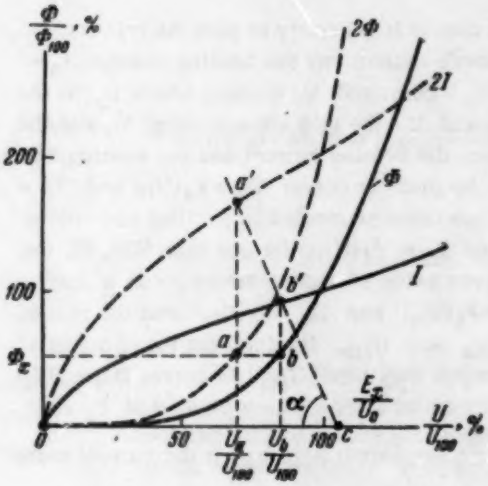


Fig. 2

In the case of the operation of lamps with photocells it is necessary to know the relationship $I_\Phi = \varphi_1(U)$, where $I_\Phi$ is the current through the photocell at a given position of the lamp and the photocell, and U is the voltage at the lamp terminals. By using relationship $I_\Phi = \varphi_1(U)$ instead of the curve $\Phi = f(U)$ we obtain in a similar manner the values of $r_0$ and $E_x$ (Fig. 3). Curve $2I_\Phi$ for the photocell current when it is lighted by two lamps can be found in various ways. Figure 4 shows the luminous characteristics of photocells with a constant color content of the luminous flux and with the voltage at the photocell $U_\Phi$ = const. In the case of a linear light characteristic of a photocell $I_\Phi = \varphi$ (U) (curve a, Fig. 4) it is possible to double the ordinates of the curve $I_\Phi = \varphi_1(U)$ which will produce the required curve $2\Phi = \varphi_1(U)$. In the case of a nonlinear light characteristic of the photocell (curve b, Fig. 4), it will be necessary to determine curve $2\Phi = \varphi_1(U)$ experimentally with a luminous flux supplied to the cell from two lamps.

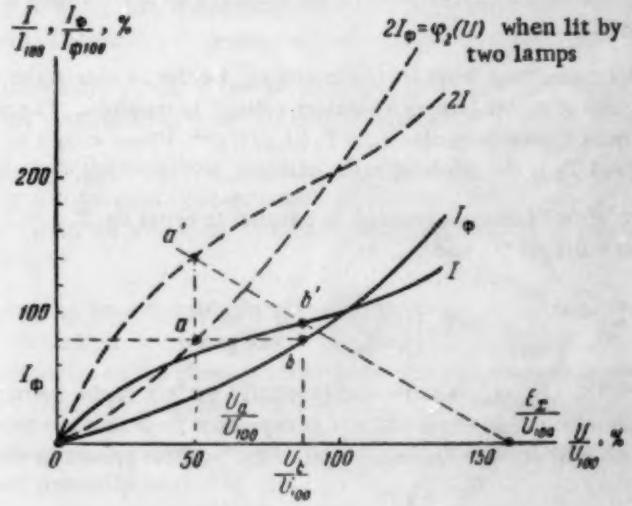


Fig. 3

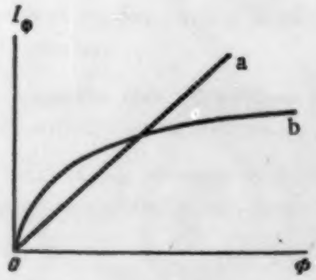


Fig. 4

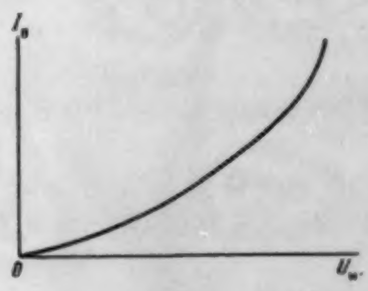


Fig. 5

Instead of obtaining the relationship $I_\Phi = \varphi_1(U)$ and $2I_\Phi = \varphi_2(U)$ experimentally, it is possible to find them by calculations.

The above technique of calculating the supply voltage and ballast resistances is also applicable to parallel connection of electron tubes.

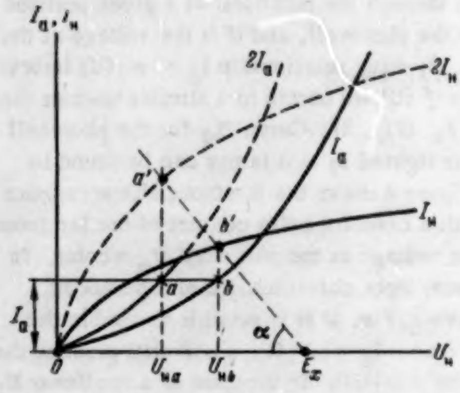


Fig. 6

In their case it is necessary to plot the relation between the anode current and the heating voltage $I_a = F_1(U_H)$ at $U_a$ = const and $U_c$ = const, where $U_a$ is the anode voltage and $U_c$ the grid voltage (Fig. 5), and the relation between the heating current and the heating voltage $I_H = f_1(U_H)$. By plotting curves $2I_a = F_2(U_H)$ and $2I_H = f_2(U_H)$ for two tubes connected in parallel and curves $I_a = F_1(U_H)$ and $I_H = f_1(U_H)$ for one tube (Fig. 6), we obtain for a given value of anode current points a and b on curves $I_a = F_1(U_H)$ and $2I_a = F_2(U_H)$ and the values of voltages $U_{Ha}$ and $U_{Hb}$. Having next found points a' and b' for voltages $U_{Ha}$ and $U_{Hb}$ on curves $2I_H$ and $I_H$, a straight line is drawn through these points, and $E_x$ and $r_0 = \dfrac{E_x - U_{Hb}}{I_{Hb}}$ are found; here $I_{Hb}$ is the current corresponding to point b'.

Depending on the matching of the characteristics $I_a = F_1(U_H)$ and $I_H = f(U_H)$ of each of the actual tubes with the corresponding calculated characteristics, the accuracy in maintaining a constant $I_a$ will be within the limits of 1.5 to 8%.

The lifetime of parallel connected tubes increases since, in order to obtain the same anode current (or luminous flux in the case of incandescent lamps) a smaller voltage is required. The relation of lifetime $T_x$ to voltage $U_x$ can be determined from formula $T_x = T_0 (U_{100}/U_x)^\alpha$ where $\alpha \approx 8$ to 12, depending on the construction of the filament, and $T_0$ is the lifetime at the nominal voltage $U_{100}$.

The combined lifetime of two lamps connected in parallel is equal to $T_{x\Sigma} = T_{xa} + T_{xb}$, where $T_{xa}$ and $T_{xb}$ are the lifetimes at voltages $U_a$ and $U_b$, or

$$T_{x\Sigma} = T_0\left[\left(\frac{U_{100}}{U_a}\right)^\alpha + \left(\frac{U_{100}}{U_b}\right)^\alpha\right].$$

The value of $\alpha$ for each tube is not the same and is usually subject to the normal distribution law. Usually the mean value is taken for $\alpha$. In some cases it is expedient to determine two lifetimes using the values $\alpha_{min}$ and $\alpha_{max}$. This will provide an evaluation of the possible spread in the lifetime.

Received February 11, 1958

## SIMPLIFIED ALGEBRAIC SYNTHESIS OF RELAY CIRCUITS

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**The paper deals with a simplified algebraic synthesis of relay circuits according to a method suitable for obtaining a structural formula directly from switching table, thus avoiding all the operations and subsequent simplifying transformations employed in normal circuit design methods. A comparison is made between this and other known methods.**

The formulation of operating conditions for various elements of multiphase circuits and the compiling of algebraic expressions which determine these conditions is accompanied by certain difficulties due to the necessity of analyzing at the same time the internal interactions in the circuits.

It is known from the theory of relay-contact circuits [1] that compiling algebraic expressions which characterize the operating conditions of a circuit and its elements can be carried out by the following methods.

1. By copying out from the switching table all the combinations of element conditions at which the given element must be in a connected (disconnected) condition. This is explained by the fact that each combination of element conditions for which the circuit of any element must be closed (open) is the condition of its operating (not operating).*

2. By means of copying from the switching table the condition of elements in the phase preceding the one in which the given element is connected and in the phase preceding the disconnecting of this element. Since the condition of elements in the phase preceding the disconnecting of this element is the condition for opening of the element's circuit the algebraic expression of this phase must be taken with an inversion.

The algebraic expressions compiled in the usual manner become complex and contain as a rule a large number of duplicated and mutually excluding circuits. It is true that the technique of the relay-contact circuit theory permits one to reduce the initial complex algebraic expressions to simpler ones, but this is panied by a complicated process of transforming the formulas originally obtained.

The method [2], worked out by M. A. Gavrilov, of dividing one unwieldy switching table into several simpler ones only consisting of elements which react on the element in question, rather simplifies the algebraic representation of the functioning of circuit elements but at the same time lengthens to a certain degree the synthesizing process.

Even when the above operations are applied the problem of compiling a circuit remains fairly complicated and requires further transformations.

The method [3], proposed by A. N. Iurasov, of obtaining a structural formula from the switching table requires subsequent complicated checking and has not, therefore, been generally adopted.

* In this instance compiling algebraic expressions for circuits based on conditions of nonfunctioning is in view.

The method described in this article is based on the fact that the elements comprising the circuits have a different effect on any one element $X_i$, depending on the position these elements occupy in the switching table. By examining the conditions under which these elements or groups of elements affect element $X_i$ it is possible to establish the rule according to which these effects are produced. The rule of interaction between elements differently placed in the switching table thus obtained permits one to avoid complicated transformations of structural formulas and to obtain algebraic expressions in the final or nearly final form directly from this table.

## 1. Schemes Requiring Discrete Circuits to Retain Blocking Arrangements Through Their Own Contacts

The functioning condition of element $X_i$ operating once per cycle is written as follows [1]:

$$fx_i = F_1 + x_i\bar{F}_2, \tag{1}$$

where $F_1$ is the Boolian function representing the product of variables which correspond to contacts of circuit elements (with the exception of element $X_i$) in the phase preceding the functioning of element $X_i$, $F_2$ is the Boolian function representing the product of variables which correspond to contacts of circuit elements (with the exception of element $X_i$) in the phase preceding the releasing of element $X_i$.

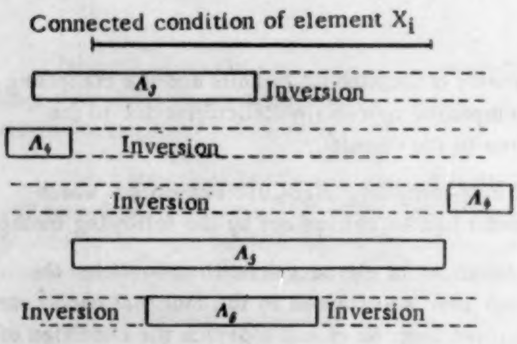



Fig. 1

Since the discrete elements of the circuit controlling element $X_i$, under consideration, occupy different positions in the switching table with respect to above element and hence affect its operation in different ways, it is expedient to divide these elements into categories with respect to the functions they fulfill (Fig. 1).*

$\Lambda_3$ is the aggregate of circuit elements which change their condition once during the interval between the 1-st and 2-nd changes in the condition of element $X_i$, under investigation;

$\Lambda_4$ is the aggregate of circuit elements which are switched in and out before the 1-st change in the condition of element $X_i$, or after the 2nd change in the condition of this element;

$\Lambda_5$ is the aggregate of circuit elements which are switched in before the 1st change in the condition of element $X_i$ and switched out after the 2nd change in the condition of the same element.

$\Lambda_6$ is the aggregate of circuit elements which change their condition twice during the interval between the 1st and 2nd changes in the condition of element $X_i$, under investigation.

The number of elements in categories $\Lambda_3$, $\Lambda_4$, $\Lambda_5$, and $\Lambda_6$ is arbitrary.

Category $\Lambda_3$ elements. If side by side with elements of other categories there is but one element of category $\Lambda_3$, the finding of the position of its contacts does not present any difficulties. In the cases when the number of elements of this category is greater than one, and in practice can be any number depending on the problem in question, it is necessary to take into consideration the order of their switching in (Fig. 2).

Let us denote by $f_3$ a Boolian function representing the product of variables which correspond to the contacts of elements of the $\Lambda_3$ category in the 1st condition

$$f_3 = x_{3\cdot 1}x_{3\cdot 2}\ldots x_{3\cdot n}, \tag{2}$$

where $x_{3\cdot 1}x_{3\cdot 2}\ldots x_{3\cdot n}$ are variables corresponding to contacts of category $\Lambda_3$ elements in the 1st condition

* In order to avoid indexing errors the numbering of $\Lambda$ subscripts follows those of F.

and placed in the switching table in the order of increasing subscripts*; $f_3$ is a Boolian function representing a product of variables which correspond to contacts of category $\Lambda_3$ elements in the 2nd condition

$$f'_3 = \bar{x}_{3\cdot1}\bar{x}_{3\cdot2}\ldots\bar{x}_{3\cdot n}. \quad (3)$$

It should be noted that $f'_3$ is a Boolian function representing the product of variables which correspond to contacts of category $\Lambda_3$ elements in the phase preceding the releasing of $X_i$.

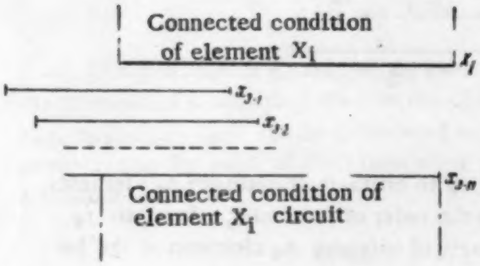



Fig. 2

The condition of contacts in the retaining circuits will be expressed by an inversion of $f'_3$, i.e.,

$$\bar{f}'_3 = x_{3\cdot1} + x_{3\cdot2} + \ldots + x_{3\cdot n}. \quad (3')$$

In the most general case elements of the $\Lambda_3$ category are connected (disconnected) in the order of increasing subscripts and disconnected (connected) in the same order.

Hence, it would appear, that the algebraic expressions of circuits consisting of these elements are not simplified by the formulas for elements functioning in a definite sequence.

If, however, the functioning of these elements is considered in conjunction with $X_i$, expression $x_ix_{3\cdot1} + x_ix_{3\cdot2} + \ldots + x_ix_{3\cdot n}$ is seen to represent a certain sequence in the functioning of these circuits, namely: simultaneous connecting, but disconnecting in the order of increasing subscripts.

Hence,

$$x_ix_{3\cdot1} + x_ix_{3\cdot2} + \ldots + x_ix_{3\cdot n} = x_ix_{3\cdot n}. \quad (4)$$

Thus, in examining circuits with more than one element of the $\Lambda_3$ category it is found that in retaining circuits there remains only one contact $x_{3\cdot n}$ which is the last one in the order of element distribution in the switching table.

Category $\Lambda_4$ elements. The Boolian function representing the product of variables which correspond to contacts of category $\Lambda_4$ elements in the phase preceding the releasing phase will take the form

$$\bar{f}_4 = \bar{x}_{4\cdot1}\bar{x}_{4\cdot2}\ldots\bar{x}_{4\cdot n}. \quad (5)$$

In this instance these elements can be omitted from the algebraic expression of the retaining circuit, since $X_i$ and $\Lambda_4$ are not connected simultaneously and therefore

$$x_ix_{4\cdot1} + x_ix_{4\cdot2} + \ldots + x_ix_{4\cdot n} = 0. \quad (5')$$

Category $\Lambda_5$ elements. The Boolian function representing the product of variables which correspond to the contacts of category $\Lambda_5$ elements in the phase preceding the releasing will take the form

$$f_5 = x_{5\cdot1}x_{5\cdot2}\ldots x_{5\cdot n}. \quad (6)$$

In this case these elements can be omitted from the algebraic expression of the retaining circuit, since $X_i$ and $\Lambda_5$ are switched in the reversed and switched out in the direct order, and hence

$$x_ix_{5\cdot1} + x_ix_{5\cdot2} + \ldots + x_ix_{5\cdot n} = 0. \quad (6')$$

* Since in the most general case the circuit contacts can be either closing or opening, their symbols in the formulas must be accompanied by the sign ~, denoting that the contact can be either one or the other. For the sake of simplifying notation this sign has been omitted.

Category $\Lambda_6$ elements. If the number of category $\Lambda_6$ elements is larger than one they can occupy in the switching table different positions with respect to element $X_i$ and to each other, for instance, in the manner shown in Fig. 3.

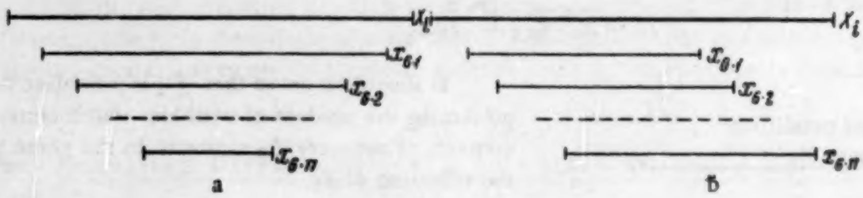



Fig. 3

Let us denote: $x_{6\cdot1}, x_{6\cdot2}, \ldots, x_{6\cdot n}$ to be variables corresponding to contacts of category $\Lambda_6$ elements, at the 1st change in condition of $X_i$, placed in the switching table in the order of increasing subscripts; $\bar{f}_6$ to be a Boolian function representing variables corresponding to contacts of category $\Lambda_6$ elements at the 1st change in condition of $X_i$:

$$\bar{f}_6 = \bar{x}_{6\cdot1}\bar{x}_{6\cdot2}\ldots\bar{x}_{6\cdot n}. \tag{7}$$

In the algebraic expression of the phase preceding the releasing, the symbols of contacts of the category under consideration are represented as multiples, i.e., the contacts in the circuit are connected in parallel.

In examining Fig. 3, a it will be seen that

$$f_6 = x_{6\cdot1} + x_{6\cdot2} + \ldots + x_{6\cdot n} = x_{6\cdot1} \tag{7'}$$

according to the principle of the equipollence of elements functioning with a given sequence, viz. connecting in the order of subscripts and disconnecting in the reversed order.

If the elements take up the position shown in Fig. 3, b where they are connected and disconnected in the order of subscripts, the algebraic expression cannot be simplified in the normal way. Below we give a method of simplifying a circuit with such a sequence of operation of its elements.

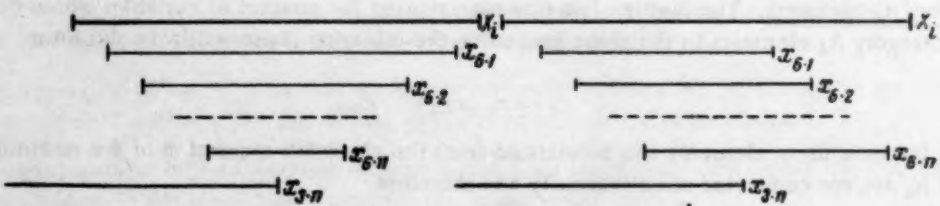



Fig. 4

It should be noted that if in a circuit controlling some element $X_i$ there are only category $\Lambda_6$ elements present, this circuit cannot be set up, in view of the presence of contradictory conditions: the circuit elements in this case must have the same conditions in the phases where $X_i$ is connected (at the beginning of connecting) and in the phase preceding the releasing, as well as in the phases of a released condition. This can be easily seen in Fig. 3.

In order to make the circuit applicable it is necessary to introduce an element of category $\Lambda_3$. The presence in the circuit of an element of this category is a necessary condition of its applicability.*

* An element of category $\Lambda_3$ is a necessary condition for setting up the retaining circuit. Since the realization of the scheme also depends on the ability to set up the connecting circuit, the above condition is necessary but not sufficient.

After inserting in the scheme an element of the category $\Lambda_3$ ($x_{3\cdot n}$) the graph under consideration will assume the form shown in Fig. 4, a. Here the elements $x_{6\cdot 1}, x_{6\cdot 2}, \ldots x_{6\cdot n}$ can be considered as being connected simultaneously and disconnected in the order of their decreasing subscripts.

The algebraic expression of the retaining circuits of such a scheme can be simplified:

$$x_i (x_{3\cdot n} + x_{6\cdot 1} + x_{6\cdot 2} + \ldots + x_{6\cdot n}) = x_i (x_{3\cdot n} + x_{6\cdot 1}). \tag{8}$$

The presence of a category $\Lambda_3$ element in the switching table, as well as making it applicable, solves the problem of simplifying the scheme of Fig. 3, b. In this case, as it will be seen in Fig. 4, b, elements $x_{6\cdot 1}, x_{6\cdot 2}, \ldots, x_{6\cdot n}$ can be considered with the presence of $f_3$ as being connected simultaneously and disconnected in the order of their increasing subscripts. The algebraic expression for a retaining circuit of such a scheme will be

$$x_i (x_{3\cdot n} + x_{6\cdot 1} + x_{6\cdot 2} + \ldots + x_{6\cdot n}) = x_i (x_{3\cdot n} + x_{6\cdot n}). \tag{8'}$$

So far only mutual positions of category $\Lambda_3$ and $\Lambda_6$ elements were considered in which the significant element of the $\Lambda_3$ category (its contact $x_{3\cdot n}$) fell within the zone occupied by all the category $\Lambda_6$ elements. It is of interest to examine the case when the element of the $\Lambda_3$ category falls in the zone occupied by only a part of the category $\Lambda_6$ elements. For instance, let us examine the graph in which contact $x_{3\cdot n}$ remains connected after the switching in of contact $x_{6\cdot 1}$ and is disconnected before the switching in of contact $x_{6\cdot 2}$ (Fig. 5).

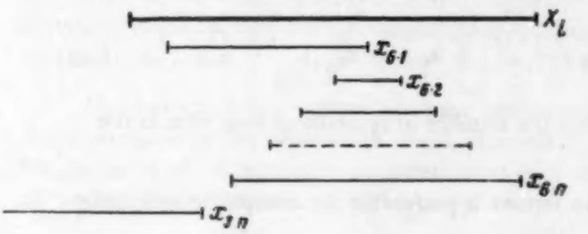


Fig. 5

In this case in a manner similar to the preceding one, the algebraic expression for the retaining circuit will be

$$x_i (x_{3\cdot n} + x_{6\cdot 1} + x_{6\cdot 2} + \ldots + x_{6\cdot n}) = x_i (x_{3\cdot n} + x_{6\cdot 1} + x_{6\cdot n}). \tag{8''}$$

It should be noted that the circuit elements can pass in the course of operation from one category to another. For instance by examining Fig. 6, a and b, which give the switching table for a consecutive operation of the circuit under the effect of multiple incoming pulses, it will be seen that the element x of category $\Lambda_3$ connects $X_i$ and then passes to category $\Lambda_6$. Figure 6, a shows a sequence of circuit operations in which the condition of contact x is the same in the phases preceding the switching in and out of $X_i$, whereas in Fig. 6, b the condition of contact x is different in the two phases. Lines on the diagram of Fig. 6 indicate a functioning condition of the appropriate circuit elements.

An algebraic expression of the circuit operating conditions with respect to both cases is given below.

Let us denote the contacts of element x when they comply with conditions of category $\Lambda_3$ by $x_{3\cdot 1}$ and when they comply with those of category $\Lambda_6$ by $x_{6_{3\cdot 1}}$.

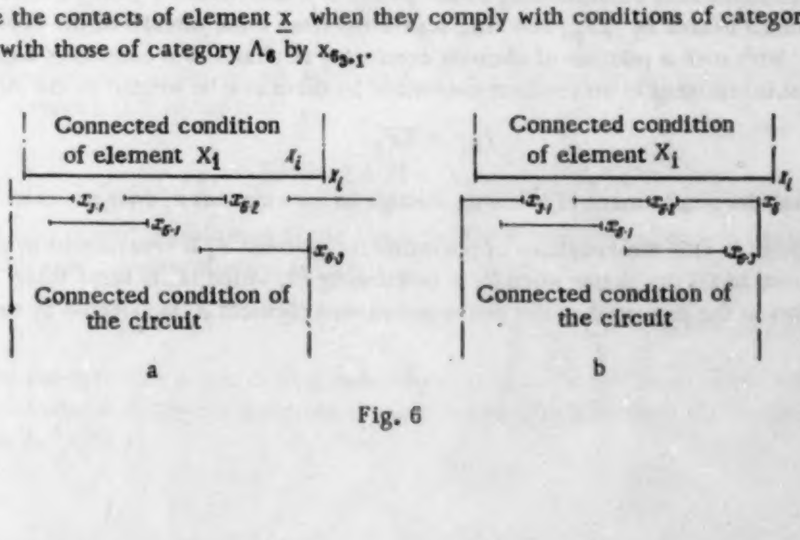


Fig. 6

According to above reasoning Expression (1) can be replaced by expression

$$
\begin{aligned}
fx_i = x_{3\cdot 1}x_{3\cdot 2}\ldots x_{3\cdot n}\overline{x}_4 x_5 \overline{x}_{6\cdot 1}\overline{x}_{6\cdot 2}\ldots \overline{x}_{6\cdot n} + \\
+ x_i(x_{3\cdot n} + x_{6\cdot 1} + x_{6\cdot 2} + \ldots + x_{6\cdot n} + x_{6_{3\cdot 1}}).
\end{aligned} \tag{9}
$$

It is known from [1] that Expression (1) can be given another form

$$fx_i = (F_1 + x_i)\overline{F}_2. \tag{10}$$

This form can be represented in another way

$$
\begin{aligned}
fx_i = F_1x_{3\cdot n} + F_1x_{6\cdot 1} + F_1x_{6\cdot 2} + \ldots + F_1x_{6\cdot n} + F_1x_{6_{3\cdot 1}} + \\
+ x_i(x_{3\cdot n} + x_{6\cdot 1} + x_{6\cdot 2} + \ldots + x_{6\cdot n} + x_{6_{3\cdot 1}}) = \\
= (F_1 + x_i)(x_{3\cdot n} + x_{6\cdot 1} + x_{6\cdot 2} + \ldots + x_{6\cdot n} + x_{6_{3\cdot 1}}).
\end{aligned} \tag{11}
$$

In Expression (11) $F_1x_{6\cdot 1}, F_1x_{6\cdot 2}, \ldots, F_1x_{6\cdot n} = 0$. If $x_{3\cdot 1} = \overline{x}_{6_{3\cdot 1}}$ then $x_{3\cdot 1}x_{6_{3\cdot 1}} = 0$. Also $F_1x_{3\cdot n} = F_1$. Let us introduce the notation $F_1' = x_{3\cdot 1}x_{3\cdot 2}\ldots x_{3\cdot(n-1)}\overline{x}_4\overline{x}_5\overline{x}_{6\cdot 1}\overline{x}_{6\cdot 2}\ldots\overline{x}_{6\cdot n}$.

Finally, we obtain

$$F = (F_1' + x_i)(x_{3\cdot n} + x_{6\cdot 1} + x_{6\cdot 2} + \ldots + x_{6\cdot n} + x_{6_{3\cdot 1}}). \tag{12}$$

In comparing Expressions (11) and (12) it will be seen that the number of symbols of elements in the latter is smaller than in the former.

Hence of the two versions $x_{3\cdot 1} = \overline{x}_{6_{3\cdot 1}}$ and $x_{3\cdot 1} = x_{6_{3\cdot 1}}$ the former is preferable for compiling switching tables.

## 2. Schemes in Which Contact Combinations in Phases With A Connected Circuit State Are Not Repeated in Phases With A Disconnected State

In deducing Formulas (9), (11), and (12) we considered the most general case according to which the setting up of the scheme in the retaining circuit with respect to any element required blocking by means of its own contact $x_i$.

Let us denote the sum of element conditions, in phases with a connected state of circuits, affecting element $X_i$ by $\Sigma F_3$ let us separate from these circuits the contacts of this element, as it was done in [2], and denote the remaining expression by $\Sigma F_3'$.

Let us examine a case when not a single combination of element contacts contained in $\Sigma F_3'$ i.e., corresponding to the connected state of circuits affecting element $X_i$, is repeated in the sum consisting of circuit-element contact-combinations corresponding to the phases of a disconnected state of circuits of the same element $X_i$, a sum we shall denote by $\Sigma F_p$, and after separation from these circuits of the contacts of element $X_i$ denote it by $\Sigma F'_p$. With such a relation of element conditions in phases of a connected and disconnected circuit the conditions of functioning of an element controlled by them can be written in the form

$$fx_i = \Sigma F_3'. \tag{13}$$

It will be seen that the requirement of blocking through its own contact $x_i$ does not exist here.

According to Expression (13) the condition of operation for element $X_i$ is represented by the sum of the circuit element conditions in all the phases when $X_i$ is functioning or, which is the same thing, by the sum of unit constituent elements in the phases when the above-mentioned element $X_i$ is affected by the functioning of other elements.

Let us denote this sum by $\Sigma K_F^{(1)}$.*

In this case Expression (13) can be written in the form [5]:

$$fx_i = \Sigma F_3' = K_F^0. \tag{13'}$$

In a manner similar to the one described above let us examine circuits $\Sigma F_3'$ which consist of various categories of circuit elements depending on their position in the switching table with respect to the element $X_i$ under consideration. Elements of categories $\Lambda_3$, $\Lambda_4$, $\Lambda_5$, and $\Lambda_6$ should be examined.

Category $\Lambda_3$, $\Lambda_4$, and $\Lambda_5$ elements. If there is only one category $\Lambda_3$ element in the circuit there is no difficulty in finding the position of its contacts in the circuit. If, however, in addition to elements of other categories there are in the circuit several elements of this category and their number can reach any value, it becomes necessary in solving this problem to take into consideration the order of their disconnecting (operation).

The symbols of category $\Lambda_3$ element contacts are contained in $\Sigma F_3'$ in the form of a sum, and the corresponding Boolian function according to the notations adopted above can be written as: $x_{3\cdot 1} + x_{3\cdot 2} + \ldots + x_{3\cdot n} = \mathcal{F}_3'$.

In the most general case it is necessary to examine elements of this category which are connected (disconnected) in the order of their increasing subscripts and disconnected (connected) in the same order, and which, it would appear, are not subject to simplification by means of formulas for elements functioning with a definite sequence.

If, however, one separates from the circuit under consideration element $X_q$ which changes its condition in the phase preceding the one in which element $X_i$ changes to its functioning condition, and then examines the operation of $\Lambda_3$ elements in conjunction with element $X_q$, one will observe a definite sequence in the operation of the circuits.

It should be noted that element $X_q$ can belong to the $\Lambda_4$, $\Lambda_5$, or $\Lambda_3$ categories.

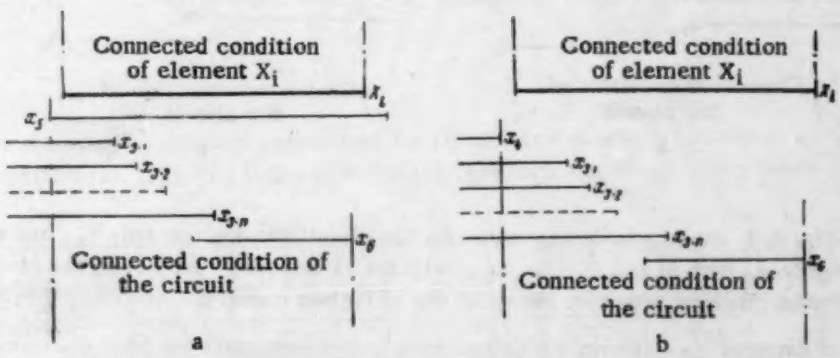



Fig. 7

If elements of categories $\Lambda_4$ and $\Lambda_5$ are taken as $X_q$ it is sufficient for the solution of the required problem to represent Expression (13) in the form

$$fx_i = \Sigma F_3' = \Sigma K_F^{(1)} = x_q \Sigma K_{F'}^{(1)} = x_q K_{F'}^{(0)}, \tag{14}$$

where $\Sigma K_{F'}^{(1)}$ is the sum of unit constituents of the phase by phase condition of circuit elements, in which $x_q$ is missing, $K_{F'}^{(0)}$ the zero constituent equal to $\Sigma K_{F'}^{(1)}$.

* The author has proved that such a sum of unit constituents is equal to the zero constituent of the elements which change their condition during the functioning condition phases of element $X_i$ in question. Let us denote this constituent by $K_F^{(0)}$.

Inspection of the graph in Fig. 7, a shows that the category $\Lambda_3$ elements considered in conjunction with $x_5$ obey a law of a definite sequence, namely simultaneous connections, but disconnections in the order of increasing subscripts

$$x_{3\cdot1}x_5 + x_{3\cdot2}x_5 + \ldots + x_{3\cdot n}x_5 = x_{3\cdot n}x_5.$$

The full condition of functioning for element $X_i$ will be according to (14):

$$f_{X_i} = x_5(x_{3\cdot n} + x_6).$$

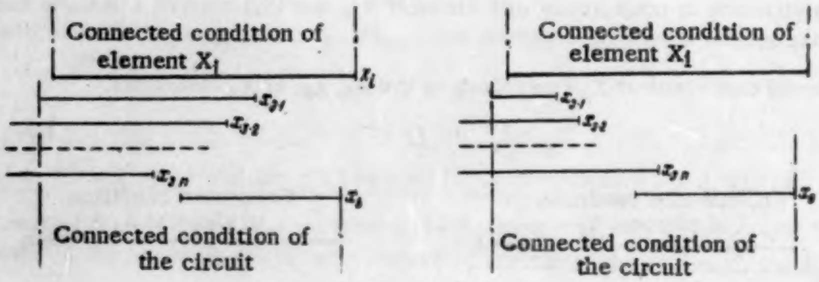
Figure 7, b shows simultaneously $\overline{x}_4$ and $x_{3\cdot1}, x_{3\cdot2}, \ldots, x_{3\cdot n}$.

Similarly to the above reasoning $x_{3\cdot1}\overline{x}_4 + x_{3\cdot2}\overline{x}_4 + \ldots + x_{3\cdot n}\overline{x}_4 = x_{3\cdot n}\overline{x}_4$ and $f_{X_i} = \overline{x}_4(x_{3\cdot n} + x_6)$.

Thus, it will be seen by inspection of circuits with category $\Lambda_3$ elements which do not change their condition during the phase preceding the change of condition in $X_i$ that the algebraic expression of the circuit operating conditions contains only one contact $x_{3\cdot n}$, which is the last in the order of contacts in the switching table.

If elements changing their condition in the phase preceding the change of condition in element $X_i$ should include category $\Lambda_3$ and not category $\Lambda_4$ or $\Lambda_5$ elements, then only one of the category $\Lambda_3$ elements will be included in the conditions of operation for element $X_i$. There will be no other elements of this category in the algebraic expression for the circuit.

This can be easily seen in Fig. 8, a which shows that the algebraic expression for the circuit is $f_{X_i} = x_{3\cdot1} + x_6$.




a b

Fig. 8

According to Fig. 8, b the algebraic expression for the circuit can include only $x_{3\cdot1}$ and $x_6$. Other elements of the category $\Lambda_3$ such as $x_{3\cdot2}, \ldots, x_{3\cdot n}$ will not be included. Moreover, the circuit cannot be set up at all without a blocking action by means of one of its own contacts.

The relation of category $\Lambda_6$ elements with elements of other categories has been discussed before. The previous statements with respect to them hold good in this case as well and do not require any additional analysis.

It was previously pointed out that circuit elements can change categories in the course of their operation. Figure 6 shows an element of category $\Lambda_3$ which switches in $X_i$ (it is denoted as $x_{3\cdot1}$) and then changes to category $\Lambda_6$ and is denoted as $x_{6_{3\cdot1}}$. Above phenomenon can occur in this case as well.

On the basis of above statements the algebraic expression of the conditions of operation for element $X_i$, written above in the form of a sum of contact conditions of elements affecting $X_i$ in all the phases of its functioning state, will be as follows.

1. If an element of the $\Lambda_4$ or $\Lambda_5$ category changes its condition in the phase preceding the functioning phase of $X_i$ under consideration:

$$f_{X_i} = \overline{x}_4x_5(x_{3\cdot n} + x_{6\cdot1} + x_{6\cdot2} + \ldots + x_{6\cdot n} + x_{6_{3\cdot1}}). \tag{15}$$

2. If an element of the $\Lambda_3$ category changes its condition in the phase preceding the functioning phase of $X_1$ under consideration:

$$fx_1 = x_3 + x_{6\cdot1} + x_{6\cdot2} + \ldots + x_{6\cdot n} + x_{6_{3\cdot1}}. \tag{15'}$$

Here $x_3$ has no subscript since it is a contact of the $\Lambda_3$ category element which changes its condition in the phase preceding the functioning phase of $X_1$ under consideration.

In deducing Formula (14), and in determining the number of element contacts of the $\Lambda_3$ category which are included in the algebraic expression of the operating conditions of $X_1$, only one element of the $\Lambda_4$ or $\Lambda_5$ category was examined. This corresponds to the conditions in a large number of applied circuits. In the most general case, however, the number of elements of the said categories can be increased indefinitely. Since these elements do not change their condition during entire period of the functioning state of $X_1$ they can be taken outside the summation sign of unity constituents and become factors of the zero constituent:

$$fx_1 = x_q\Sigma K_F^{(1)} = \bar{x}_{4\cdot1}\bar{x}_{4\cdot2}\ldots\bar{x}_{4\cdot n}x_{5\cdot1}x_{5\cdot2}\ldots x_{5\cdot n}\Sigma K_F^{(1)} =$$
$$= x_{4\cdot1}x_{4\cdot2}\ldots x_{4\cdot n}x_{5\cdot1}x_{5\cdot2}\ldots x_{5\cdot n}K_F^{(0)},$$

where $\Sigma K_F^{(1)}$ is the sum of unity constituents of the phase-by-phase condition of circuit elements which lack all the elements of the $\Lambda_4$ and $\Lambda_5$ categories; and $K_F^{(0)}$ is the zero constituent equal to $\Sigma K_F^{(1)}$.

The algebraic expression of the functioning conditions of element $X_1$ can be written for the most general case in the following form:

$$fx_1 = \Pi\bar{x}_4\Pi x_5(x_{3\cdot n} + x_{6\cdot1} + x_{6\cdot2} + \ldots + x_{6\cdot n} + x_{6_{3\cdot1}}).$$

Moreover, if in the phase preceding the functioning phase of $X_1$ under consideration an element of the $\Lambda_4$ or $\Lambda_5$ category changes its condition, then $x_{3\cdot n}$ is the element contact of the $\Lambda_3$ category, which is the last in the order of their position in the switching table.

If, however, one of the elements of category $\Lambda_3$ changes its condition then $x_{3\cdot n}$ will be the contact of that very element.

## 3. Examples

Examples of setting up operating conditions for all circuit elements by means of an algebraic method according to Formulas (9), (11), and (12) and according to other methods are given below for comparison purposes.

| Phases | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Binary number | $-a$ | $+a$ | | $-a$ | | $+a$ | | $-a$ | | $+a$ | | $-a$ | | $+a$ | | $-a$ | |
| | $-X_1$ | | $+X_1$ | | | | $-X_1$ | | | | | | | | | | |
| | $-X_2$ | | | | $+X_2$ | | | | | | | | $-X_2$ | | | | |
| | $-X_3$ | | | | | | | | $+X_3$ | | | | | | | | $-X_3$ |
| | $-X_4$ | | | | | | | | | | $+X_4$ | | | | $-X_4$ | | |

Fig. 9

Let us inspect the switching table given in Fig. 9 and determine conditions of operation of circuit elements by former methods described in [1] and [4] and by the proposed method according to Formulas (9) and (12).

Above examples of comparison between circuit synthesis by the ordinary methods and the method proposed in this paper show the advantage of the latter. This method does not require complicated transformations and provides results directly from the switching table.

| Methods according to [1 and 4] | Methods according to Formulas (9) and (12) |
|---|---|
| 1) $fX_1 = a\bar x_2\bar x_3\bar x_4 + x_1\overline{a\bar x_2\bar x_3\bar x_4} = a\bar x_2\bar x_3\bar x_4 + x_1(\bar a + \bar x_2 + \bar x_3 + \bar x_4)$<br>Since $X_1$ and $X_3$ as well as $X_1$ and $X_4$ are not connected simultaneously<br>$x_1x_3 = 0$ и $x_1x_4 = 0$<br>$fX_1 = a\bar x_2\bar x_3\bar x_4 + x_1(\bar a + \bar x_2)$<br>or<br>$fX_1 = a\bar x_2\bar x_3\bar x_4 + a\bar x_3\bar x_4\bar a + x_1(\bar a + \bar x_2) = (a\bar x_3\bar x_4 + x_1)(\bar a + \bar x_2)$ | 1) $f_3 = a,\ \bar x_2;\ f_4 = \bar x_3,\ \bar x_4;\ f_{6_{s-1}} = \bar a$<br>According to (9)<br>$fX_1 = a\bar x_2\bar x_3\bar x_4 + x_1(\bar a + \bar x_2)$<br>or according to (12)<br>$fX_1 = (a\bar x_3 x_4 + x_1)(\bar a + \bar x_2)$ |
| 2) $fX_2 = \bar a x_1\bar x_3\bar x_4 + x_2\overline{a x_1 x_3 x_4} = \bar a x_1\bar x_3\bar x_4 + x_2(a + x_1 + \bar x_3 + \bar x_4)$<br>Since $x_2x_1 + x_2\bar x_3 + x_2\bar x_4 = x_2\bar x_4$<br>(circuits connected with a definite sequence) we have<br>$fX_2 = \bar a x_1\bar x_3\bar x_4 + x_2(a + \bar x_4)$<br>or<br>$fX_2 = \bar a x_1\bar x_3\bar x_4 + \bar a x_1\bar x_3 a + x_2(a + \bar x_4) = (\bar a x_1\bar x_3 + x_2)(a + \bar x_4)$ | 2) $f_3 = \bar a,\ x_1,\ \bar x_3,\ \bar x_4;\ f_{6_{s-1}} = a$<br>According to (9)<br>$fX_2 = \bar a x_1\bar x_3\bar x_4 + x_2(a + \bar x_4)$<br>According to (12)<br>$fX_2 = (\bar a x_1\bar x_3 + x_2)(a + \bar x_4)$ |
| 3) $fX_3 = \bar a\bar x_1 x_2\bar x_4 + x_3\overline{a x_1 x_2 x_4} = \bar a\bar x_1 x_2\bar x_4 + x_3(a + x_1 + x_2 + x_4)$<br>Since $X_1$ and $X_3$ are not connected simultaneously $x_1x_3 = 0$.<br>$fX_3 = \bar a\bar x_1 x_2 x_4 + x_3(a + x_2 + x_4)$<br>or<br>$fX_3 = \bar a\bar x_1 x_2\bar x_4 + \bar a\bar x_1\bar x_4 a + \bar a\bar x_1\bar x_4 x_4 + x_3(a + x_2 + x_4) = (\bar a\bar x_1\bar x_4 + x_3)(a + x_2 + x_4)$ | 3) $f_3 = a,\ x_2;\ f_4 = \bar x_1;\ f_6 = x_4;\ f_{6_{s-1}} = a$<br>According to (9)<br>$fX_3 = a\bar x_1 x_2\bar x_4 + x_3(a + x_2 + x_4)$<br>According to (12)<br>$fX_3 = (a\bar x_1\bar x_4 + x_3)(a + x_2 + x_4)$ |
| 4) $fX_4 = a\bar x_1 x_2 x_3 + x_4\overline{a\bar x_1 x_2 x_3} = a\bar x_1 x_2 x_3 + x_4(\bar a + x_1 + x_2 + \bar x_3)$<br>Since $X_1$ and $X_4$ are not connected simultaneously $x_1x_4 = 0$.<br>Since $X_3$ and $X_4$ are connected in the order of increasing subscripts but are disconnected in the reverse order<br>$\bar x_3 x_4 = 0$<br>$fX_4 = a\bar x_1 x_2 x_3 + x_4(\bar a + x_2)$<br>or<br>$fX_4 = a\bar x_1 x_2 x_3 + a\bar x_1 x_3\bar a + x_4(\bar a + x_2) + (a\bar x_1 x_3 + x_4)(\bar a + x_2)$ | 4) $f_3 = a,\ x_2;\ f_4 = \bar x_1;\ f_5 = x_3;\ f_{5_{s-1}} = \bar a$<br>According to (9)<br>$fX_4 = a x_2\bar x_1 x_3; + x_4(\bar a + x_2)$<br>According to (12)<br>$fX_4 = (a\bar x_1 x_3 + x_4)(\bar a + x_2)$ |

It has been stated previously that in addition to the above method there are other methods which do not require copying the structural formulas for all the elements contained in the switching table, but only require determining the elements which are in the circuits of the element whose conditions of operation are being determined.

Among such methods is the one proposed by M. A. Gavrilov [2] which consists in dividing the basic switching tables into auxiliary ones.

The previously used examples are now solved by means of compiling new switching tables, only containing elements which change their positions in the phases preceding the functioning or releasing of the element under consideration and then checking the applicability of the tables thus obtained.

The new switching tables are given in Figs. 10, a ; 11, a; 12, a and 13, a. It will be seen that not one of the newly compiled tables is applicable, and according to the rules of the method it is now necessary to find elements in the general table which would make the tables applicable, and add these elements to each new table.

| Phases | Row | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Binary Nos. | $2^\circ$ | $-a$ | $+a$ | | $-a$ | | $+a$ | | $-a$ | | $+a$ | | $-a$ | | $+a$ | | $-a$ | |
| | $2'$ | $-X_1$ | | $+X_1$ | | | | $-X_1$ | | | | | | | | | | |
| Sum of combinations | | 0 | 1 | 3 | 2 | | 3 | 1 | 0 | | 1 | | 0 | | 1 | | 0 | |

a

| Phases | Row | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Binary Nos. | $2^\circ$ | $-a$ | $+a$ | | $-a$ | | $+a$ | | $-a'$ | | $+a$ | | $-a$ | | $+a$ | | $-a$ | |
| | $2'$ | $-X_1$ | | $+X_1$ | | | | $-X_1$ | | | | | | | | | | |
| | $2^2$ | $-X_2$ | | | | $+X_2$ | | | | | | | | $-X_2$ | | | | |
| Sum of combinations | | 0 | 1 | 3 | 2 | 6 | 7 | 5 | 4 | | 5 | | 4 | 0 | 1 | . | 0 | |

b

| Phases | Row | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Binary Nos. | $2^\circ$ | $-a$ | $+a$ | | $-a$ | | $+a$ | | $-a$ | | $+a$ | | $-a$ | | $+a$ | | $-a$ | |
| | $2'$ | $-X_1$ | | $-X_1$ | | | | $-X_1$ | | | | | | | | | | |
| | $2^2$ | $-X_2$ | | | | $+X_2$ | | | | | | | | $-X_2$ | | | | |
| | $2^3$ | $-X_3$ | | | | | | | | $+X_3$ | | | | | | | | $-X_3$ |
| Sum of combinations | | 0 | 1 | 3 | 2 | 6 | 7 | 5 | 4 | 12 | 13 | | 12 | 8 | 9 | | 8 | 0 |
| Sum of combinations w/o $X_1$ | | 0 | 1 | 1 | 0 | 4 | 5 | 5 | 4 | 12 | 13 | | 12 | 8 | 9 | | 8 | 0 |

c

Fig. 10

| Phases | Row | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Binary Nos. | $2^0$ | $-a$ | $+a$ | | $-a$ | | $+a$ | | $-a$ | | $+a$ | | $-a$ | | $+a$ | | $-a$ | |
| | $2'$ | $-X_2$ | | | | $+X_2$ | | | | | | | | $-X_2$ | | | | |
| Sum of combinations | | 0 | 1 | | 0 | 2 | 3 | | 2 | | 3 | | 2 | 0 | 1 | | 0 | |

a

| Phases | Row | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Binary Nos. | $2^0$ | $-a$ | $+a$ | | $-a$ | | $+a$ | | $-a$ | | $+a$ | | $-a$ | | $+a$ | | $-a$ | |
| | $2'$ | $-X_1$ | | $+X_1$ | | | | $-X_1$ | | | | | | | | | | |
| | $2^2$ | $X_2$ | | | | $+X_2$ | | | | | | | | $-X_2$ | | | | |
| Sum of combinations | | 0 | 1 | 3 | 2 | 6 | 7 | 5 | 4 | | 5 | | 4 | 0 | 1 | | 0 | |

b

| Phases | Row | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Binary Nos. | $2^0$ | $-a$ | $+a$ | | $-a$ | | $+a$ | | $-a$ | | $+a$ | | $-a$ | | $+a$ | | $-a$ | |
| | $2'$ | $-X_1$ | | $+X_1$ | | | | $-X_1$ | | | | | | | | | | |
| | $2^2$ | $-X_2$ | | | | $+X_2$ | | | | | | | | $-X_2$ | | | | |
| | $2^3$ | $-X_4$ | | | | | | | | | | $+X_4$ | | | | $-X_4$ | | |
| Sum of combinations | | 0 | 1 | 3 | 2 | 6 | 7 | 5 | 4 | | 5 | 13 | 12 | 8 | 9 | 1 | 0 | |
| Sum of combinations w/o $X_1$ | | 0 | 1 | 3 | 2 | 2 | 3 | 1 | 0 | | 1 | 9 | 8 | 8 | 9 | 8 | 0 | |

c

Fig. 11

| Phases | Row | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Binary Nos. | $2^0$ | $-a$ | $+a$ | | $-a$ | | $+a$ | | $-a$ | | $+a$ | | $-a$ | | $+a$ | | $-a$ | |
| | $2'$ | $-X_3$ | | | | | | | | $+X_3$ | | | | | | | | $-X_3$ |
| Sum of combinations | | 0 | 1 | | 0 | | 1 | | 0 | 2 | 3 | | 2 | | 3 | | 2 | 0 |

a

| Phases | Row | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Binary Nos. | $2^0$ | $-a$ | $+a$ | | $-a$ | | $+a$ | | $-a$ | | $+a$ | | $-a$ | | $+a$ | | $-a$ | |
| | $2'$ | $-X_1$ | | $+X_1$ | | | | $-X_1$ | | | | | | | | | | |
| | $2^2$ | $-X_3$ | | | | | | | | $+X_3$ | | | | | | | | $-X_3$ |
| Sum of combinations | | 0 | 1 | 3 | 2 | | 3 | 1 | 0 | 4 | 5 | | 4 | | 5 | | 4 | 0 |

b

| Phases | Row | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Binary Nos. | $2^0$ | $-a$ | $+a$ | | $-a$ | | $+a$ | | $-a$ | | $+a$ | | $-a$ | | $+a$ | | $-a$ | |
| | $2'$ | $-X_1$ | | $+X_1$ | | | | $-X_1$ | | | | | | | | | | |
| | $2^2$ | $-X_3$ | | | | | | | | $+X_3$ | | | | | | | | $-X_3$ |
| | $2^3$ | $-X_4$ | | | | | | | | | | $+X_4$ | | | | $-X_4$ | | |
| Sum of combinations | | 0 | 1 | 3 | 2 | | 3 | 1 | 0 | 4 | 5 | 13 | 12 | | 13 | 5 | 4 | 0 |

c

| Phases | Row | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Binary Nos. | $2^0$ | $-a$ | $+a$ | | $-a$ | | $+a$ | | $-a$ | | $+a$ | | $-a$ | | $+a$ | | $-a$ | |
| | $2'$ | $-X_1$ | | $+X_1$ | | | | $-X_1$ | | | | | | | | | | |
| | $2^2$ | $-X_2$ | | | | $+X_2$ | | | | | | | | $-X_2$ | | | | |
| | $2^3$ | $-X_3$ | | | | | | | | $+X_3$ | | | | | | | | $-X_3$ |
| | $2^4$ | $-X_4$ | | | | | | | | | | $+X_4$ | | | | $-X_4$ | | |
| Sum of combinations | | 0 | 1 | 3 | 2 | 6 | 7 | 5 | 4 | 12 | 13 | 29 | 28 | 24 | 25 | 9 | 8 | 0 |

d

Fig. 12

| Phases | Row | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Binary Nos. | $2^0$ | $-a$ | $+a$ | | $-a$ | | $+a$ | | $-a$ | | $+a$ | | $-a$ | | $+a$ | | $-a$ | |
| | $2'$ | $-X_4$ | | | | | | | | | | $+X_4$ | | | | $-X_4$ | | |
| Sum of combinations | | 0 | 1 | | 0 | | 1 | | 0 | | 1 | 3 | 2 | | 3 | 1 | 0 | |

a

| Phases | Row | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Binary Nos. | $2^0$ | $-a$ | $+a$ | | $-a$ | | $+a$ | | $-a$ | | $+a$ | | $-a$ | | $+a$ | | $-a$ | |
| | $2'$ | $-X_3$ | | | | | | | | $+X_3$ | | | | | | | | $-X_3$ |
| | $2^2$ | $-X_4$ | | | | | | | | | | $+X_4$ | | | | $-X_4$ | | |
| Sum of combinations | | 0 | 1 | | 0 | | 1 | | 0 | 2 | 3 | 7 | 6 | | 7 | 3 | 2 | 0 |

b

| Phases | Row | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Binary Nos. | $2^0$ | $-a$ | $+a$ | | $-a$ | | $+a$ | | $-a$ | | $+a$ | | $-a$ | | $+a$ | | $-a$ | |
| | $2'$ | $-X_2$ | | | | $+X_2$ | | | | | | | | $-X_2$ | | | | |
| | $2^2$ | $-X_3$ | | | | | | | | $+X_3$ | | | | | | | | $-X_3$ |
| | $2^3$ | $-X_4$ | | | | | | | | | | $+X_4$ | | | | $-X_4$ | | |
| Sum of combinations | | 0 | 1 | | 0 | 2 | 3 | | 2 | 6 | 7 | 15 | 14 | 12 | 13 | 5 | 4 | 0 |
| Sum of combinations w/o $X_1$ | | 0 | 1 | | 0 | 2 | 3 | | 2 | 6 | 7 | | 6 | 4 | 5 | | 4 | 0 |

c

Fig. 13

Such elements were found to be the following: $X_2$ and $X_3$ in obtaining operating conditions for element $X_1$; $X_1$ and $X_4$ for element $X_2$; $X_1$, $X_2$, and $X_4$ for element $X_3$; and $X_2$ and $X_3$ for element $X_4$.

The determining of the additional contacts does not present any difficulties and is shown in Figs. 10-13. The final switching table for element $X_1$ is given in Fig. 10, c, for element $X_2$ in Fig. 11, c; for element $X_3$ in Fig. 12, d and for element $X_4$ in Fig. 13, c.

Thus, the operating conditions of the circuit elements are determined as follows:

$$f_{X_1} = a\bar{x}_2\bar{x}_3 + x_1 a\bar{x}_3 = a\bar{x}_2\bar{x}_3 + x_1(\bar{a} + \bar{x}_3),$$

$$f_{X_2} = \bar{a}x_1\bar{x}_4 + x_2\bar{\bar{a}}x_4 = \bar{a}x_1\bar{x}_4 + x_2(a + \bar{x}_4),$$

$$f_{X_3} = \bar{a}x_1x_2\bar{x}_4 + x_3\overline{\bar{a}\bar{x}_2\bar{x}_4} = \bar{a}x_1x_2\bar{x}_4 + x_3(a + x_2 + x_4).$$

$$f_{X_4} = ax_2x_3 = x_4\bar{\bar{a}}x_2 = ax_2x_3 + x_4(\bar{a} + x_2).$$

It is useful to inspect the switching table given in [2] and to compile operating conditions for circuit elements by the method recommended here according to Formulas (9) and (15) and to compare it with M. A. Gavrilov's method.

The full switching table is given in Fig. 14.

| Phases | Row | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $2^0$ | $-a$ | $+a$ | | | | | | | | | |
| Binary Nos. | $2'$ | $-X_1$ | | $+X_1$ | | | | | $-X_1$ | | | $+X_1$ |
| | $2^2$ | $-X_2$ | | | $+X_2$ | | | | | $-X_2$ | | |
| | $2^3$ | $-X_3$ | | | | $+X_3$ | | $-X_3$ | | | | |
| | $2^4$ | $-X_4$ | | | | | $+X_4$ | | | | $-X_4$ | |
| Sum of combinations | | 0 | 1 | 3 | 7 | 15 | 31 | 23 | 21 | 17 | 1 | 3 |

Fig. 14

It is shown in [2] that according to M. A. Gavrilov's method in order to determine the elements affecting respectively $X_1$, $X_2$, $X_3$, and $X_4$, it is necessary to compile tables which form parts of the table in Fig. 14. It is also stated there that in determining operating conditions for each element these tables can turn out to be invalid, as is the case in determining the conditions of operation for $X_1$. This fact leads to the re uirement of inserting an additional element, that is, to compiling a new table. The determination of the operating conditions for elements $X_1$, $X_2$, $X_3$, and $X_4$ by this method is given in [2].

Let us determine the conditions of operation of the same elements $X_1$, $X_2$, $X_3$, and $X_4$ by the proposed method according to Formulas (9) and (15).

The conditions of operation for element $X_1$ are determined by Formula (15): $f_3 = \bar{x}_2\bar{x}_4$. $f_5 = a$; $f_6 = x_3$. $fX_1 = a(\bar{x}_4 + x_3)$.

The conditions of operation for element $X_2$ are: $f_3 = \bar{x}_4$, $x_1$. $f_6 = x_3$. According to Formula (15) $fX_2 =$ $= x_1$.

The conditions of operation for element $X_3$ are: $f_3 = \bar{x}_4$. $f_5 = x_2$. According to Formula (15) $fX_3 = x_2\bar{x}_4$.

The conditions of operation for element $X_4$ are: $f_4 = x_3$, $x_1$, $x_2$. According to Formula (9) $fX_4 =$ $= ax_1x_2x_3 + x_4x_2$.

Taking into consideration formulas for a given sequence applied to connecting circuits (the retaining circuits obtained by Formula (9) are not subject to simplification) we obtain $f_{X_4} = ax_3 + x_4x_2$.

## SUMMARY

1. The method described in the paper lends itself to a rapid calculation of algebraic expressions for circuits direct from a switching table without all the operations and the accompanying simplifying transformations which are employed in the existing methods of setting up circuits.

2. M. A. Gavrilov's method, based on the division of the switching table into several simpler ones without changing the principle of setting up the circuit, simplifies the synthesizing process but at the same time rather lengthens it.

3. In comparing M. A. Gavrilov's method with the one described in this paper, it should be noted that the determination of the connecting circuits is accomplished by the former quicker than by the latter. It is recommended, however, to carry out the determination of the retaining circuits by the method described in this paper.

## LITERATURE CITED

[1] M. A. Gavrilov, The Theory of Relay Contact Circuits (AN SSSR Press, 1950).

[2] M. A. Gavrilov, "Basic formulas for synthesizing relay circuits," Avtomatika i Telemekhanika 15, 6 (1954).

[3] A. N. Iurasov, Setting up Structural Formulas for Multiphase Circuits [In Russian] Moscow Mechanical Institute, (Mashgiz, M., 1952).

[4] Ia. I. Makler, Constituents Sufficient for Ensuring the Operation of Relay-Contact Circuits (Trans. AN SSSR 117, 4, 1957).

Received June 8, 1958

# AN OPERATIONAL AMPLIFIER WITH A DIFFERENTIAL INPUT

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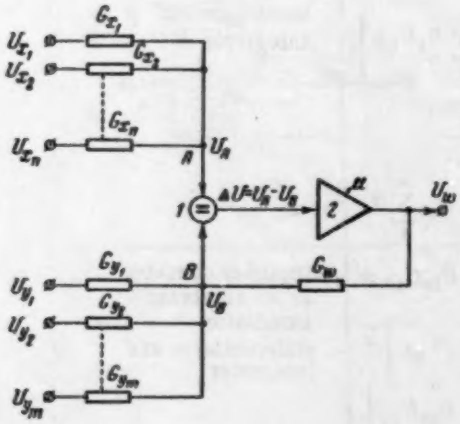
(Leningrad)

The paper describes block schematics of amplifiers capable of simulating, due to specially designed input and feedback circuits, the greater part of integro-differential relationships important from the practical point of view.

Operational dc amplifiers with a compensation circuit (see sketch) can be used for solving electrical simulation problems.




Operational amplifiers operating on this principle were proposed by Philbrick in 1949 and form the basic component of an electronic analog made by a U. S. firm of the same name. These amplifiers are capable of simulating, due to the differential input circuit, a wide range of functions. Some important practical applications of these amplifiers produce simpler circuit designs than conventional Soviet made amplifiers used in electronic simulating devices.

In order to demonstrate the possibilities of an operational amplifier with a differential input let us determine the relationship between its output voltage $U_w$, the input voltage $U_{xk}$ and the conductances $G_{xk}$ and $G_{yk}$.

From inspection of the block schematic it immediately follows that the output voltage $U_W$ is the root of equation

$$U_A - U_B = \frac{U_w}{\mu}.$$

A substitution of junction voltages $U_A$ and $U_B$ by input voltages $U_{xk}$ and $U_{yk}$ and the output voltage determines the implicit function

$$U_w = \frac{\mu\left[\sum_1^n U_{xk} h_{xk}(p) - \sum_1^m U_{yk} h_{yk}(p)\right]}{1 + \mu h_w(p)},$$

where

$$h_{xk}(p) = \frac{G_{xk}(p)}{\sum_1^n G_{xk}(p)}, \quad h_{yk}(p) = \frac{G_{yk}(p)}{G_w(p) + \sum_1^m G_{yk}(p)}$$

and transfer functions of the corresponding input circuits, and $h_w(p) = 1 - \sum_1^m h_{yk}(p)$

TABLE

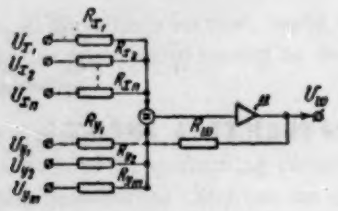
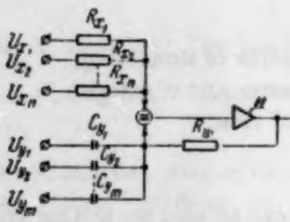
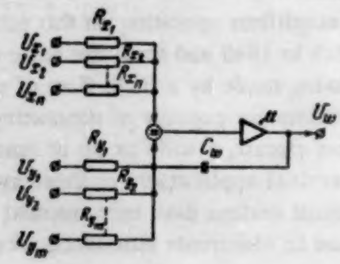
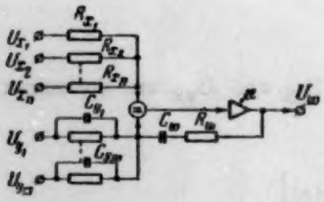
| Amplifier block schematic | Simulated relationship $U_w=U_w[U_{xk},\ U_{yk},\ h_{xk}(p),\ h_{yk}(p)]$ | Remarks |
|---|---|---|
|  | $U_w=\sum_1^n A_k U_{xk}-\sum_1^m B_k U_{ky}$, $A_k=\dfrac{1+R_w\sum_1^m \dfrac{1}{R_{yk}}}{R_{xk}\sum_1^n \dfrac{1}{R_{kx}}}$, $B_k=\dfrac{R_w}{R_{yk}}$ | Amplifier operates as an algebraic summator |
|  | $U_w=\sum_1^n A_{1k}U_{xk}+$ $+p\left[\sum_1^n A_{2k}U_{xk}-\sum_1^m B_k U_{yk}\right]$, $A_{1k}=\dfrac{1}{R_{xk}\sum_1^n \dfrac{1}{R_{xk}}}$, $B_k=R_w C_{yk},\quad A_{2k}=A_{1k}\sum_1^m B_k$ | Amplifier operates as an algebraic summator and differentiator |
|  | $U_w=\sum_1^n A_{1k}U_{xk}+$ $+p^{-1}\left[\sum_1^n A_{2k}U_{xk}-\sum_1^m B_k U_{yk}\right]$, $A_{1k}=\dfrac{1}{R_{xk}\sum_1^n \dfrac{1}{R_{xk}}}$, $B_k=\dfrac{1}{C_w R_{yk}},\ A_{2k}=A_{1k}\sum_1^m B_k$ | Amplifier operates as an algebraic summator and integrator |
|  | $U_w=\sum_1^n A_{1k}U_{xk}-\sum_1^m B_{1k}U_{yk}+$ $+p\left[\sum_1^n A_{2k}U_{xk}-\sum_1^m B_{2k}U_{yk}\right]+$ $+p^{-1}\left[\sum_1^n A_{3k}U_{xk}-\sum_1^m B_{3k}U_{yk}\right]$, $A_{1k}=\dfrac{1}{R_{xk}\sum_1^n \dfrac{1}{R_{xk}}}\left(1+\sum_1^n \dfrac{C_{yk}}{C_w}+\right.$ $\left.+\sum_1^m \dfrac{R_w}{R_{yk}}\right),\ B_{1k}=\dfrac{C_{yk}}{C_w}+\dfrac{R_w}{R_{yk}}$, $A_{2k}=\dfrac{1}{R_{xk}\sum_1^n \dfrac{1}{R_{xk}}}\sum_1^m R_w C_{yk}$, $B_{2k}=R_w C_{yk},\ A_{3k}=\dfrac{1}{R_{xk}\sum_1^n \dfrac{1}{R_{xk}}}\times$ $\times\sum_1^m \dfrac{1}{C_w R_{yk}},\ B_{3k}=\dfrac{1}{C_w R_{yk}}$ | Amplifier operates as an algebraic summator, differentiator and integrator |

TABLE (Continued)

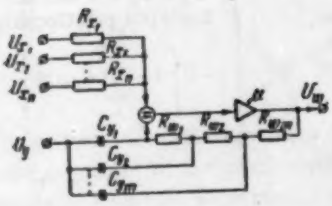
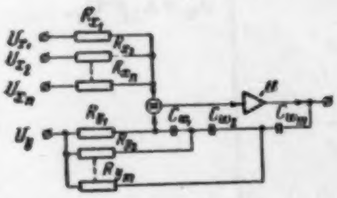
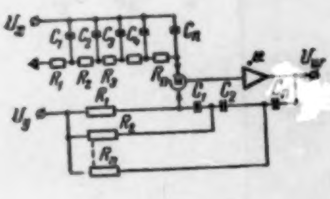
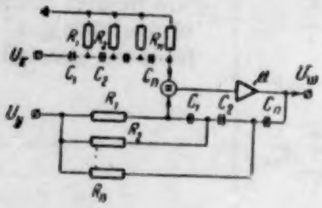
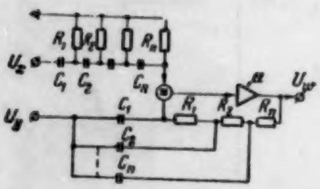
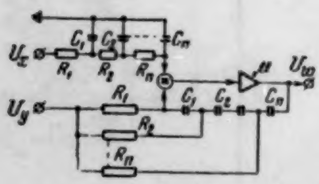
| Amplifier block schematic | Simulated relationship $U_w - U_w[U_{xk}, U_{yk}, h_{xk}(p), h_{yk}(p)]$ | Remarks |
|---|---|---|
|  | $U_w = \sum_1^n A_{1k} U_{xk}\left[1+\sum_1^m A_{2k}p^k\right] - U_y \sum_1^m A_{2k}p^k$, $h_{xk} = \frac{1}{R_{xk}\sum_1^n \frac{1}{R_{xk}}}$ $(k=1, 2, \dots, n)$, $h_y = \frac{\sum_1^m A_{2k}p^k}{1+\sum_1^m A_{2k}p}$ $h_w = \frac{1}{1+\sum_1^m A_{2k}p^k}$ | Algebraic summator and differentiator of the m-th order ($h_{xk}$, $h_y$, and $h_w$ are transfer functions of the input and feedback circuits) |
|  | $U_w = \sum_1^n A_{1k} U_{kx}\left[1+\sum_1^m A_{2k}p^{-k}\right] - U_y \sum_1^m A_{2k}p^{-k}$, $h_{xk} = \frac{1}{R_{xk}\sum_1^n \frac{1}{R_{xk}}}$ $(k=1, 2, \dots, n)$, $h_y = \frac{1+\sum_1^{m-1} B_kp^k}{1+\sum_1^m B_kp^k}$, $h_w = \frac{B_mp^m}{1+\sum_1^m B_kp^k}$ | Algebraic times m summator and integrator ($h_{xk}$, $h_y$ and $h_w$ are transfer functions of the input and feedback circuits) |
|  | $U_w = U_x\left[1+\sum_1^{n-1} A_kp^{-k}\right] - U_y \sum_1^n A_kp^{-k}$, $h_x = \frac{\sum_1^n A_kp^k}{1+\sum_1^n A_kp^k}$, $h_w = \frac{A_np^n}{1+\sum_1^n A_kp^k}$, $h_y = \frac{1+\sum_1^{n-1} A_kp^k}{1+\sum_1^n A_kp^k}$ | Algebraic summator and integrator; with $U_x = U_y = U$ the simulated relationship is $U_w = U(1 - A_np^{-n})$ |

TABLE (Continued)

| Amplifier block schematic | Simulated relationship $U_w = U_w[U_{xk}, U_{yk}, h_{xk}(p), h_{yk}(p)]$ | Remarks |
|---|---|---|
|  | $U_w = U_x - U_y \sum_1^n A_k p^{-k},$<br>$h_x = \frac{A_n p^n}{1 + \sum_1^n A_k p^k},$<br>$h_w = \frac{A_n p^n}{1 + \sum_1^n A_k p^k},$<br>$h_y = \frac{1 + \sum_1^{n-1} A_k p^k}{1 + \sum_1^n A_k p^k}$ | Algebraic summator and integrator; with $U_x = U_y = U$ the simulated relationship is $U_w = U\left[1 - \sum_1^n A_k p^{-k}\right]$ |
|  | $U_w = U_x A_n p^n - U_y \sum_1^n A_k p^k,$<br>$h_x = \frac{A_n p^n}{1 + \sum_1^n A_k p^k},$<br>$h_w = \frac{1}{1 + \sum_1^n A_k p^k},$<br>$h_y = \frac{\sum_1^n A_k p^k}{1 + \sum_1^n A_k p^k}$ | Algebraic summator and differentiator; with $U_y = 0$ the simulated relationship is $U_w = A_n p^n U_x$. |
|  | $U_w = A_n p^{-n} U_x - U_y \sum_1^n A_k p^{-k}$<br>$h_x = \frac{1}{1 + \sum_1^n A_k p^k},$<br>$h_w = \frac{A_n p^n}{1 + \sum_1^n A_k p^k},$<br>$h_y = \frac{1 + \sum_1^{n-1} A_k p^k}{1 + \sum_1^n A_k p^k}$ | Algebraic summator and integrator; with $U_y = 0$ the simulated relationship is $U_w = A_n p^{-n} U_x$. |

is the transfer function of the feedback circuit.

When condition $|\mu h_w(p)| \gg 1$ is fulfilled the final expression for the amplifier output voltage will take the form

$$U_w = \frac{\sum_1^n U_{xk} h_{xk}(p) - \sum_1^m U_{yk} h_{yk}(p)}{h_w(p)}.$$

It follows from the last expression that for input voltages $U_{xk}$ the amplifier acts as a repeater and does not improve the quality of the passive summation network $U_A$, but for voltages $U_{yk}$ the amplifier represents a normal anode to grid feedback circuit and considerably improves the quality of the summation network $U_B$.

This circumstance must be taken into account when the amplifier is used in electronic simulating devices.

The shape of the simulating characteristic of $U_w$ is determined by the type of conductances $G(p)$ and the way they are connected in the input and output circuits of the amplifier.

The table shows the most typical practical schematic circuits of this amplifier for simulating automatic control systems we well as relationships corresponding to these circuits.

The accuracy of simulating relationships given in the table is determined not only by the theoretical error

$$[\delta U_w]_t = \frac{100}{\mu}\left[1 + \left|\frac{\sum_1^m G_{yk}}{G_w}\right|\right]\%,$$

but also by the instrument error, due to the difference in the amplification factors $\mu_1$ and $\mu_2$ of the first differential stage of the amplifier. This error is given by the formula

$$[\Delta U_w]_i = \frac{1}{h_w(p)}\left[\frac{\Delta\mu_1}{\mu_2} - \frac{\mu_1}{\mu_2}\Delta\mu_2\right]\sum_1^n U_{xk} h_{xk}(p),$$

where $\Delta\mu_1$ and $\Delta\mu_2$ are quantities representing the instability of the amplification factors of the first stage tubes.

At $\mu_1 = \mu_2$

$$[\delta U_w]_i = \left|\frac{\sum_1^n U_{xk} h_{xk}(p)}{\sum_1^n U_{xk} h_{xk}(p) - \sum_1^m U_{yk} h_{yk}(p)}\right| (\delta_{\mu_1} - \delta_{\mu_2})\,\%.$$

## SUMMARY

1. An operational amplifier with a differential input is a universal device capable of simulating relationships (algebraic summation, integration, differentiation and other more complex integro-differential functions) without changing the sign of the input voltage. As a result of this property it becomes possible to omit in analog circuits special sign-changing (inversion) operations which require additional amplifiers with a $\mu = -1$.

2. The accuracy of the described amplifier is a little lower than that of conventional operational amplifiers due to the error in obtaining the voltage difference in the differential stage of the amplifier. However, the schematic simulation of the majority of function given in the table by means of conventional amplifiers is more complicated. Hence, the possible decrease in accuracy when a differential input amplifier is used should be considered on the merits of each particular case.

3. With zero input voltages $U_{xk} = 0$ the amplifier operates as a conventional anode to grid feedback amplifier.

Received October 23, 1957

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Compiled by G. Moskatov

## FIRST IFAC CONGRESS

**The International Federation on Automatic Control (IFAC) will hold its first Congress in Moscow in 1960.**

**The USSR National Committee on Automatic Control (Moscow, 15 a Kalanchevskaia Street) invites all specialists engaged in scientific and practical work in the sphere of automation to prepare for this Congress.**

**Persons wishing to read papers at the Congress should apply to the National Committee, enclosing the title of the paper with a brief annotation, before February 15, 1959.**

**The scientific program of the Congress and its organizational details will be published in the journal, Avtomatika i Telemekhanika No. 1, 1959.**

**USSR National Committee on**
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| | |
|---|---|
| **AN SSSR** | *Academy of Sciences, USSR* |
| **FIAN** | *Physics Institute, Academy of Sciences USSR* |
| **GITI** | *State Scientific and Technical Press* |
| **GITTL** | *State Press for Technical and Theoretical Literature* |
| **GOI** | *State Optical Institute* |
| **GONTI** | *State United Scientific and Technical Press* |
| **Gosenergoizdat** | *State Power Press* |
| **Gosfizkhimizdat** | *State Physical Chemistry Press* |
| **Gozkhimizdat** | *State Chemistry Press* |
| **GOST** | *All-Union State Standard* |
| **Goztekhizdat** | *State Technical Press* |
| **GTTI** | *State Technical and Theoretical Press* |
| **GUPIAE** | *State Office for Utilization of Atomic Energy* |
| **IF KhI** | *Institute of Physical Chemistry Research* |
| **IFP** | *Institute of Physical Problems* |
| **IL** | *Foreign Literature Press* |
| **IPF** | *Institute of Applied Physics* |
| **IPM** | *Institute of Applied Mathematics* |
| **IREA** | *Institute of Chemical Reagents* |
| **ISN (Izd. Sov. Nauk)** | *Soviet Science Press* |
| **IIaP** | *Institute of Nuclear Studies* |
| **Izd** | *Press (publishing house)* |
| **KISO** | *Solar Research Commission* |
| **LETI** | *Leningrad Electrotechnical Institute* |
| **LFTI** | *Leningrad Institute of Physics and Technology* |
| **LIM** | *Leningrad Institute of Metals* |
| **LITMiO** | *Leningrad Institute of Precision Instruments and Optics* |
| **Mashgiz** | *State Scientific-Technical Press for Machine Construction Literature* |
| **MATI** | *Moscow Aviation Technology Institute* |
| **MGU** | *Moscow State University* |
| **Metallurgizdat** | *Metallurgy Press* |
| **MOPI** | *Moscow Regional Pedagogical Institute* |
| **NIAFIZ** | *Scientific Research Association for Physics* |
| **NIFI** | *Scientific Research Institute of Physics* |
| **NIIMM** | *Scientific Research Institute of Mathematics and Mechanics* |
| **NII ZVUKSZAPIOI** | *Scientific Research Institute of Sound Recording* |
| **NIKFI** | *Scientific Institute of Motion Picture Photography* |
| **OIIaI** | *Joint Institute of Nuclear Studies* |
| **ONTI** | *United Scientific and Technical Press* |
| **OTI** | *Division of Technical Information* |
| **OTN** | *Division of Technical Science* |
| **RIAN** | *Radium Institute, Academy of Sciences of the USSR* |
| **SPB** | *All-Union Special Planning Office* |
| **Stroiizdat** | *Construction Press* |
| **URALFTI** | *Ural Institute of Physics and Technology* |

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